1. (8 points each) Evaluate the following integrals. Simplify your answer in parts (a)–(c).

(a) \[ \int_{0}^{1} 9x^2 (x^3 + 1)^2 \, dx. \]

Let \( u = x^3 + 1 \)
\[ du = 3x^2 \, dx \]

Then, the integral becomes
\[ \int_{1}^{2} 3u^2 \, du = \left[ u^3 \right]_1^2 = 8 - 1 = 7 \]

(b) \[ \int_{0}^{3} 8e^{2x} \, dx. \]

Let \( u = 2x \)
\[ du = 2 \, dx \]

Then, the integral becomes
\[ \int_{0}^{6} 4e^u \, du = \left[ 4e^u \right]_0^6 = 4e^6 - 4 \]
(c) \[ \int_0^{\pi/3} 12 \cos^2 x \sin x \, dx = \int_{-1}^{\frac{1}{4} \sqrt{3}} \frac{1}{2} \, du = \left[ -4u^2 \right]^{\frac{1}{2}} = -\frac{1}{2} + 4 = 2 \]

(d) \[ \int \frac{x}{x^2 + 9} \, dx = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln (x^2 + 9) + C \]
(e) \[ \int \frac{4}{\sqrt{1-x^2}} \, dx = 4 \sin^{-1} x + C \]

(f) \[ \int \tan^5 x \sec^2 x \, dx = \int u^5 \, du = \frac{1}{6} u^6 + C \]

\[ u = \tan x \]
\[ du = \sec^2 x \, dx \]
\[ = \frac{1}{6} \tan^6 x + C \]
2. (5 points each) Evaluate the following integrals.

(a) \[ \int \frac{e^{3x}}{1 + e^{6x}} \, dx = \int \frac{e^{3x}}{1 + (e^{3x})^2} \, dx = \frac{1}{3} \tan^{-1}(e^{3x}) + C \]

(b) \[ \int \frac{3x}{\sqrt{x+4}} \, dx = \int \frac{3(u-4)}{u} \, du = (3u - 12) \, du = 3u^{3/2} - 24u^{1/2} + C = 2(x+4)^{3/2} - 24(x+4)^{1/2} + C \]
3. (7 points each) Set up, but do not evaluate, integrals for each of the following quantities.

(a) The area of the region on the interval $[1, 5]$ which is bounded below by the $x$-axis and above by the graph of $y = \frac{1}{1 + x^2}$.

\[
\text{AREA} = \int_1^5 \frac{1}{1 + x^2} \, dx
\]

(b) The volume obtained when the region in part (a) is revolved around the $x$-axis.

\[
VOLUME = \int_1^5 \pi \left( \frac{1}{1 + x^2} \right)^2 \, dx
\]

(c) The volume obtained when the region in part (a) is revolved around the $y$-axis.

\[
VOLUME = \int_1^5 2\pi x \cdot \frac{1}{1 + x^2} \, dx
\]
(d) The average value of the function $f(x) = \ln x$ over the interval $[1, 3]$.

\[
\text{AVERAGE} = \frac{1}{3-1} \int_{1}^{3} \ln x \, dx
\]

\[
= \frac{1}{2} \int_{1}^{3} \ln x \, dx
\]

(e) The length of the curve $f(x) = \ln x$ from $x = 1$ to $x = 5$.

\[
\text{LENGTH} = \int_{1}^{5} \sqrt{1 + \left(\frac{1}{x}\right)^2} \, dx
\]
4. (7 points) An inverted conical tank has a 2 foot radius at the top and is 6 feet high. It is filled to a height of 5 feet with olive oil weighing 57 lb/ft³. Set up, do not evaluate, a definite integral which represents the amount of work that it takes to pump the oil to a point 2 feet above the top rim of the tank.

\[
\text{WORK} = \int_{0}^{5} 57\pi \left(\frac{1}{3}y\right)^{3} (8-y) \, dy
\]