1. (6 points) Sketch a rough graph for the function $f(x) = \frac{32}{2^x}$ on the interval [0, 6] and shade in the area between the $x$-axis and the function on this interval.

(a) Approximate the shaded area using a right Riemann sum with 3 rectangles. Simplify your answer. Is there any reason to believe that your approximation is an underestimate or an overestimate of the exact shaded area? Explain or indicate with a picture.

(b) Approximate the shaded area using a left Riemann sum with 3 rectangles. Simplify your answer. Is there any reason to believe that your approximation is an underestimate or an overestimate of the exact shaded area? Explain or indicate with a picture.

(c) Approximate the shaded area by averaging your two answers above. Simplify your answer.
2. (6 points) A car is traveling at 80 feet per second when the driver sees a deer in the road 350 feet ahead and immediately steps on the brakes. The deer freezes and does not move from his spot in the road. I’ve recorded the driver’s speed (in ft/sec) every two seconds starting at the time that he first stepped on the brakes and going until the time that the car finally came to a stop. From the moment the driver steps on the brakes, determine both an underestimate and overestimate of the distance the car travels before coming to a stop. You may assume that the car is always slowing down until it comes to a stop. Does the car hit the deer?

<table>
<thead>
<tr>
<th>time</th>
<th>car’s speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

3. (5 points) Find a function $f$ such that $f'(x) = 3e^{2x}$ and such that $f(0) = \frac{1}{2}$. 


4. (5 points) Find a function \( f \) such that \( f''(x) = 6x \) and such that \( f(1) = 10 \) and \( f'(1) = 7 \).

5. (5 points) The graph of \( f(x) \) is shown below. If \( \int_{4}^{6} f(x) \, dx = 8 \), then evaluate the following definite integrals.

(a) \( \int_{6}^{4} f(x) \, dx \)

(b) \( \int_{3}^{3} f(x) \, dx \)

(c) \( \int_{2}^{0} f(x) \, dx \)

(d) \( \int_{6}^{6} f(x) \, dx \)

(e) \( \int_{6}^{4} f(x) \, dx \)
6. (6 points) Find the area between the graph of \( y = 8e^{-2x} \) and the \( x \)-axis on the interval \([0, 1]\).

7. (6 points each) Evaluate the following indefinite integrals.

(a) \( \int \left(10x^5 + x^2 + 7\right) \, dx \)
(b) $\int \frac{3x^2 + 1}{6x^3} \, dx$

(c) $\int 4x \left( x^2 + 5x + 2 \right)^3 \, dx$
(d) $\int \sin^5 x \cos^3 x \, dx$

(e) $\int \frac{6}{9 + 4x^2} \, dx$
8. (7 points each) Evaluate the following definite integrals.

\begin{align*}
(f) & \int x \sqrt{x + 5} \, dx \\
(a) & \int_{2}^{3} \left( x^2 - 4x \right) \, dx
\end{align*}
(b) \[ \int_{0}^{2} \frac{1}{e^x} \, dx \]

(c) \[ \int_{0}^{1} (2x + 1)^3 \, dx \]
(d) \[ \int_{\pi/6}^{\pi/4} 8 \sec^2 x \tan^3 x \, dx \]

9. (3 points) Evaluate the following indefinite integral.
\[ \int \frac{x + 25}{x + 10} \, dx \]