Name __SOLUTIONS__

- No calculators are allowed.
- Show sufficient work to justify your answer.

1. (5 points) Find $f'(x)$ given that $f(x) = \left(x^3 + 4x^2 - 3x + 2\right)^5$

\[
f'(x) = 5\left(x^3 + 4x^2 - 3x + 2\right)^4 \cdot (3x^2 + 8x - 3)
\]

2. (5 points) Find $h'(x)$ given that $h(x) = \ln \left(\sin \left(\sqrt{x}\right)\right)$

\[
h'(x) = \frac{1}{\sin \left(\sqrt{x}\right)} \cdot \cos \left(\sqrt{x}\right) \cdot \frac{1}{2} x^{-1/2}
\]
3. (5 points) Find \( \frac{dy}{dx} \) given that \( y = \sec^{-1} x \)

\[
\begin{align*}
\sec y &= x \\
\sec y \tan y \frac{dy}{dx} &= 1 \\
\frac{dy}{dx} &= \frac{1}{x \sqrt{x^2 - 1}}
\end{align*}
\]

**or more generally**

\[
\frac{dy}{dx} = \frac{1}{x^2 \sqrt{x^2 - 1}}
\]

4. (10 points) Find \( f'(x) \) given that \( f(x) = x^3 \ln x \)

\[
\begin{align*}
f'(x) &= 3x^2 \ln x + x^3 \cdot \frac{1}{x} \\
f'(x) &= 3x^2 \ln x + x^2
\end{align*}
\]
5. (10 points) Find \( \frac{dw}{dx} \) given that \( w = \tan(e^{5x}) \)

\[
\frac{dw}{dx} = 5 \sec^2(e^{5x}) \cdot e^{5x} \cdot 5
\]

6. (10 points) Find \( g'(x) \) given that \( g(x) = \tan^{-1}(3x) \)

\[
g'(x) = \frac{1}{1 + (3x)^2} = \frac{1}{1 + 9x^2}
\]

Or could derive as in #3
7. (10 points) Find $\frac{dy}{dx}$ given that $xy + y^2 = 2$. Your answer may be written in terms of both $x$ and $y$.

\[
\frac{d}{dx}(xy + y^2) = \frac{d}{dx}(2)
\]

\[
\frac{d}{dx}(xy) + y + x \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = 0
\]

\[
1 \cdot y + x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0
\]

\[
(x + 2y) \frac{dy}{dx} = -y
\]

\[
\frac{dy}{dx} = \frac{-y}{x + 2y}
\]

8. (10 points) The point $(0, 2)$ is on the graph of the curve given by the equation $e^x + y^3 = y + 7$. Find the slope of the tangent line at this point.

\[
\frac{d}{dx}(e^x + y^3) = \frac{d}{dx}(y + 7)
\]

\[
e^x + 3y^2 \frac{dy}{dx} = \frac{dy}{dx}
\]

Letting $x = 0$ and $y = 2$ we get

\[
e^0 + 3(2)^2 \frac{dy}{dx} = \frac{dy}{dx}
\]

\[
1 + 12 \frac{dy}{dx} = \frac{dy}{dx}
\]

\[
11 \frac{dy}{dx} = -1
\]

\[
\frac{dy}{dx} = -\frac{1}{11}
\]
9. (10 points) A television camera is positioned 4000 feet from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Suppose the rocket rises vertically and its speed is 600 feet per second when it has risen 3000 feet. How fast is the camera’s angle of elevation changing at that moment?

\[
\text{GIVEN: } \frac{dh}{dt}\bigg|_{h=3000 \text{ ft}} = 600 \text{ ft/sec}
\]

\[
\text{FIND: } \frac{d\theta}{dt}\bigg|_{h=3000 \text{ ft}}
\]

\[
\tan \theta = \frac{h}{4000}
\]

\[
\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{4000} \frac{dh}{dt}
\]

\[
\left(\frac{5}{4}\right)^2 \frac{d\theta}{dt} = \frac{1}{4000} (600)
\]

\[
\frac{25}{16} \frac{d\theta}{dt} = 6
\]

\[
\frac{d\theta}{dt} = \frac{6 \cdot 16}{25}
\]

\[
\frac{d\theta}{dt} = \frac{96}{25}
\]

\[
\frac{d\theta}{dt} = 0.096 \text{ radians/sec}
\]
10. (5 points) Find a formula for \( f^{-1}(x) \) given that \( f(x) = 2x^3 + 5 \).

\[
\begin{align*}
  y &= 2x^3 + 5 \\
  y - 5 &= 2x^3 \\
  \frac{y - 5}{2} &= x^3 \\
  x &= \sqrt[3]{\frac{y - 5}{2}} \\
  y &= x^3 \quad \text{(switch x & y)} \\
  f^{-1}(x) &= \frac{3}{2}x - \frac{5}{2}
\end{align*}
\]

11. (5 points) Complete the identity below so that the right side of your equation does not have any trigonometric or inverse trigonometric functions.

\[
\sin \left(2\sin^{-1} x\right) = \sin (2\theta)
\]

Let \( \theta = \sin^{-1} x \)

\[
\begin{align*}
  \sin \theta &= x \\
  \frac{1}{\sqrt{1-x^2}} &= \cos \theta \\
  \sin (2\theta) &= 2 \times \sqrt{1-x^2}
\end{align*}
\]
12. (5 points) Carefully sketch a graph of either \( y = \sec x \) or \( y = \tan^{-1} x \).

13. (5 points) Find a formula for \( \frac{dy}{dx} \) given that \( y = x^{2x} \)

\[
\ln y = \ln (x^{2x})
\]

\[
\ln y = 2x \ln x
\]

\[
\frac{1}{y} \frac{dy}{dx} = 2 \ln x + 2 \cdot \frac{1}{x}
\]

\[
\frac{dy}{dx} = y \left( 2 \ln x + 2 \right)
\]

\[
\frac{dy}{dx} = x^{2x} \left( 2 \ln x + 2 \right)
\]
14. (5 points) Wheat is poured through a chute at the rate of 10 \text{ ft}^3/\text{min}, and falls in a conical pile whose bottom radius is always half the altitude. How fast will the circumference of the base be increasing when the pile is 8 \text{ ft} high?

\[ V = \frac{1}{3} \pi r^2 h \]

\[ V = \frac{1}{3} \pi r^2 (2r) \]

\[ V = \frac{2}{3} \pi r^3 \]

\[ V = \frac{2\pi}{3} \left( \frac{C}{2\pi} \right)^3 \]

\[ V = \frac{1}{12\pi^2} C^3 \]

\[ \frac{dV}{dt} = \frac{1}{12\pi^2} \cdot 3C^2 \frac{dc}{dt} \]

\[ 10 = \frac{1}{4\pi^2} \cdot (8\pi)^2 \frac{dc}{dt} \]

SINCE \ h=8 \Rightarrow r=4 \Rightarrow C=8\pi

\[ \frac{dc}{dt} = \frac{5}{8} \text{ ft/ min} \]

or \[ \frac{dc}{dt} = 0.625 \text{ ft/ min} \]