Math 141 (Section 501)  Test 4  November 18, 2004

Name ________________________________

- No calculators are allowed.

1. (9 points) Evaluate the following indefinite integrals.

(a) \( \int x^r \, dx \) \quad (r \neq -1)

(b) \( \int \frac{1}{x} \, dx \)

(c) \( \int \cos x \, dx \)

(d) \( \int \sin x \, dx \)

(e) \( \int \sec^2 x \, dx \)

(f) \( \int \sec x \tan x \, dx \)

(g) \( \int e^x \, dx \)

(h) \( \int \frac{1}{1 + x^2} \, dx \)

(i) \( \int \frac{1}{\sqrt{1 - x^2}} \, dx \)
2. (6 points) Evaluate the following indefinite integral.
\[
\int \left( 5x^4 + 3x^{-1} + 22 \right) \, dx
\]

3. (6 points) Evaluate the following indefinite integral.
\[
\int \frac{\tan^2 x}{\sin^2 x} \, dx
\]
4. (8 points) Evaluate the following indefinite integral.
\[ \int 2x \cos (4x^2 + 3) \, dx \]

5. (6 points) Evaluate the following indefinite integral.
\[ \int \frac{6}{1 + 9x^2} \, dx \]
6. (8 points) Precisely state the text of either the Mean-Value Theorem or Rolle’s Theorem. Be sure to indicate which theorem you are stating.

7. (4 points) Suppose that \( f(x) = \frac{24(x - 2)(x^2 + 3)(x^4 + 4)(x - 5)}{100(x^2 + 6)(x^4 + 7)} \). Between which two \( x \)-values does the above theorem guarantee the existence of stationary points. Recall that stationary points are \( x \)-values for which \( f'(x) = 0 \).
8. (10 points) Evaluate the following sum and simplify your answer.

\[ \sum_{k=1}^{100} (2k + 7) \]

9. (6 points) Write the following sum using sigma notation, but do not evaluate.

\[ 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \cdots + \frac{1}{25^2} - \frac{1}{26^2} + \frac{1}{27^2} \]
10. (7 points) For $-2 < x < 2$, answer the following questions based upon the graph of $f(x)$ shown below.

(a) Is $f(x)$ positive, negative, or zero?
(b) Is $f(x)$ increasing, decreasing, or constant?
(c) Is $f(x)$ concave up, concave down, or neither?
(d) Is $f'(x)$ positive, negative, or zero?
(e) Is $f'(x)$ increasing, decreasing, or constant?
(f) Is $f''(x)$ positive, negative, or zero?

(g) Which one of the following three statements is correct?
   i. $f(x)$ is increasing at an increasing rate.
   ii. $f(x)$ is increasing at a decreasing rate.
   iii. $f(x)$ is increasing at a constant rate.

11. (14 points) Let $f(x) = x^3 - 9x^2 + 24x$.

(a) Find the $x$-values for all stationary points on the graph of $f(x)$. State whether or not each of these $x$-values will yield a relative maximum, a relative minimum, or neither.

(b) Find the $x$-values for all inflection points on the graph of $f(x)$. 


12. (8 points) A dairy farmer plans to fence in a rectangular pasture adjacent to a river. She figures that the pasture must contain 180,000 $m^2$ in order to provide enough grass for her herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

13. (8 points) Who do you expect to win tomorrow’s game? *Hint:* You must say USC if you want these 8 points!