- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- No calculators allowed!
- Show your Student ID when you turn in your test.

---

<table>
<thead>
<tr>
<th>#1 (6 points)</th>
<th>#11 (6 points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#2 (5 points)</td>
<td>#12 (5 points)</td>
</tr>
<tr>
<td>#3 (6 points)</td>
<td>#13 (6 points)</td>
</tr>
<tr>
<td>#4 (5 points)</td>
<td>#14 (6 points)</td>
</tr>
<tr>
<td>#5 (5 points)</td>
<td>#15 (6 points)</td>
</tr>
<tr>
<td>#6 (5 points)</td>
<td>#16 (6 points)</td>
</tr>
<tr>
<td>#7 (5 points)</td>
<td>#17 (6 points)</td>
</tr>
<tr>
<td>#8 (5 points)</td>
<td>#18 (6 points)</td>
</tr>
<tr>
<td>#9 (5 points)</td>
<td></td>
</tr>
<tr>
<td>#10 (6 points)</td>
<td></td>
</tr>
</tbody>
</table>

Total (100 points)
1. (6 points) [simplify your answer] If \( g(t) = 20t^{10} - 20t^3 + 9 \), then
\[
g'(t) = 200t^9 - 60t^2 \quad \text{or} \quad 20t^2(10t^7 - 3)
\]
\[
g'(t) = 20(10t^7) - 20(3t^3) + 0
\]
\[
= 200t^9 - 60t^2
\]

2. (5 points) If \( h(x) = 5 - \ln x \), then
\[
h'(x) = -\frac{1}{x}
\]
\[
h'(x) = 0 - \frac{1}{x}
\]

3. (6 points) If \( y = 8^t + e^t \), then
\[
\frac{dy}{dt} = \ln 8 \cdot 8^t + e^t
\]
4. (5 points) [Simplify your answer] If \( y = \frac{-1}{3x^3} \), then

\[
\frac{dy}{dx} = \frac{\ast \times -4}{\ast \times 1} \quad \text{or} \quad \frac{1}{x^4}
\]

\[
y = -\frac{1}{3} x^{-3}
\]

\[
\frac{dy}{dx} = -\frac{1}{3} \left(-3x^{-4}\right)
\]

\[
= \ast x^{-4}
\]

5. (5 points) If \( y = \sqrt{x} \), then

\[
\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \quad \text{or} \quad \frac{1}{2 \sqrt{x}}
\]

\[
y = x^{\frac{1}{2}}
\]

\[
\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}}
\]

6. (5 points) If \( w = e^{-5t} \), then

\[
\frac{dw}{dt} = -5 e^{-5t}
\]

\[
\frac{dw}{dt} = e^{-5t} \cdot (-5t)'
\]

\[
= e^{-5t} \cdot (-5)
\]

\[
= -5 e^{-5t}
\]
7. (5 points) [simplify your answer] If \( P(t) = 10t^5 \ln t \), then

\[
P'(t) = 50t^4 \ln t + 10t^4
\]

\[
P'(t) = (10t^5)'(\ln t) + (10t^5)(\ln t)'
\]

\[
= 50t^4 \ln t + 10t^5 \left( \frac{1}{t} \right)
\]

\[
= 50t^4 \ln t + 10t^4
\]

or

\[
= 10t^4 (5 \ln t + 1)
\]

8. (5 points) If \( y = \frac{t^3 + 2}{t^2 + 1} \), then

\[
\frac{dy}{dt} = \frac{3t^2(t^2+1) - (t^3+2)(2t)}{(t^2+1)^2}
\]

\[
\frac{dy}{dt} = \frac{(t^3+2)'(t^2+1) - (t^3+2)(t^2+1)'}{(t^2+1)^2}
\]

\[
= \frac{3t^2(t^2+1) - (t^3+2)(2t)}{(t^2+1)^2}
\]

or

\[
= \frac{t^4 + 3t^2 - 4t}{t^4 + 2t^2 + 1}
\]
9. (5 points) [simplify your answer] If \( y = (e^{3\ln x})^2 \), then

\[
\frac{dy}{dx} = 6x^5
\]

**Short Way**

\[
y = (e^{3\ln x})^2 = (e^{\ln(x^3)})^2 = (x^3)^2 = x^6
\]

So \( y' = 6x^5 \)

**Long Way**

\[
y' = 2(e^{3\ln x}) \cdot e^{3\ln x} \cdot \frac{3}{x} = 2(e^{\ln x^3}) \cdot (e^{\ln x^3}) \cdot \frac{3}{x} = 2(x^3)(x^3) \cdot \frac{3}{x} = 6x^5
\]

10. (6 points) Ralph has a man-made pond in his backyard. The pond had no fish so he purchased some guppies. They reproduced many times and Ralph noted that the total number of guppies could be approximated by the function \( g(t) = t^2 + 50 \), where \( t \) represents the number of months since his original purchase. Precisely five months after his original purchase, the total number of guppies in his pond are increasing by

(a) 10 guppies per month
(b) 15 guppies per month
(c) 20 guppies per month
(d) 50 guppies per month
(e) 70 guppies per month
(f) 100 guppies per month
(g) 150 guppies per month

\[ g'(t) = 2t \]
\[ g'(5) = 10 \]
11. (6 points) If \( f(x) = \ln(5x^3) \), then what is the value of \( f'(2) \)?

(a) \( \ln(2) \)

(b) \( \frac{1}{\ln(2)} \)

(c) \( \ln(40) \)

(d) \( \frac{1}{\ln(40)} \)

(e) \( \frac{1}{2} \)

(f) \( \frac{1}{5} \)

(g) \( \frac{3}{2} \)

(h) 0

(i) 1

(j) \( e^{40} \)

(l) \( \frac{1}{e^{40}} \)

\[
\begin{align*}
f'(x) &= \frac{1}{5x^3} \cdot 15x^2 \\
f'(x) &= \frac{3}{x} \\
f'(2) &= \frac{3}{2}
\end{align*}
\]

12. (5 points) State whether the derivative of the function graphed below is positive, negative, or zero at each of the labeled points.

[Diagram of a function graph with labeled points A, B, C, D, E, and the axes labeled x and f(x).]
13. (6 points) Find the equation of the line tangent to the graph of $f(x) = x^2 + 5x + 3$ at $x = 1$.

(a) $y = 7x + 2$
(b) $y = 7x + 3$
(c) $y = 7x + 5$
(d) $y = 7x + 9$
(e) $y = 5x + 2$
(f) $y = 5x + 3$
(g) $y = 5x + 5$
(h) $y = 5x + 9$
(i) $y = 2x + 2$
(j) $y = 2x + 3$
(k) $y = 2x + 5$
(l) $y = 2x + 9$

Function $f(x) = x^2 + 5x + 3$ at $x = 1$:
$f(1) = 9$
$50$ point $= (1, 9)$

$f'(x) = 2x + 5$
$50$ slope $= 1.5$
$f'(1) = 7$

Equation:
$y - 9 = 7(x - 1)$
$y = 7x + 2$

14. (6 points) Below is a graph of the function $g(x) = \frac{\ln x}{x}$. What are the coordinates $(x, y)$ for the local maximum value shown? Be sure each coordinate is in simplified form.

\[ g'(x) = \frac{(\ln x)'(x) - (\ln x)(x)'}{x^2} \]
\[ g'(x) = \frac{x - \ln x - 1}{x^2} \]
\[ g'(x) = \frac{1 - \ln x}{x^2} \]
\[ 0 = \frac{1 - \ln x}{x^2} \]
\[ 0 = 1 - \ln x \]
\[ \ln x = 1 \]
\[ x = e \]
\[ g(e) = \frac{\ln e}{e} = \frac{1}{e} \]
15. (6 points) A function \( f(x) \) is given below along with its first and second derivatives in factored and unfactored forms.

- \( f(x) = x^4 - 4x^3 + 16x - 16 = (x + 2)(x - 2)^3 \)
- \( f'(x) = 4x^3 - 12x^2 + 16 = 4(x + 1)(x - 2)^2 \)
- \( f''(x) = 12x^2 - 24x = 12x(x - 2) \)

The graph of \( f(x) \) is decreasing upon which one of the following intervals?

(a) \([-2, 2]\]  
(b) \([-1, 2]\]  
(c) \([0, 2]\]  
(d) \((\infty, -2]\)  
(e) \((\infty, -1]\)  
(f) \((\infty, 0]\)  
(g) \([-2, \infty)\]  
(h) \([-1, \infty)\]  
(i) \([0, \infty)\]  
(j) \((\infty, \infty)\]

16. (6 points) A function \( g(x) \) has the following derivative.

\[ g'(x) = 5e^x(x - 1)^2(x - 2)^3(x - 3)^4 \]

Which one of the following statements is true about the graph of \( g(x) \) ?

(a) There is a local minimum at \( x = 3 \)
(b) There is a local minimum at \( x = 2 \)
(c) There is a local minimum at \( x = 1 \)
(d) There is a local minimum at \( x = 0 \)
(e) There is a local minimum at \( x = -1 \)
(f) There is a local maximum at \( x = 3 \)
(g) There is a local maximum at \( x = 2 \)
(h) There is a local maximum at \( x = 1 \)
(i) There is a local maximum at \( x = 0 \)
(j) There is a local maximum at \( x = -1 \)
17. (6 points) A ball is tossed straight up with an initial velocity of 16 feet per second. The ball is 8 feet above the ground when it is released. Its height at time $t$ is given by

$$h = -16t^2 + 16t + 8$$

How high does it go before returning to the ground?

(a) 5 feet
(b) 6 feet
(c) 8 feet
(d) 9 feet
(c) 12 feet
(f) 16 feet
(g) 24 feet
(h) 32 feet

$$h' = -32t + 16$$
$$0 = -32t + 16$$
$$32t = 16$$
$$t = \frac{1}{2}$$

$$h\left(\frac{1}{2}\right) = -16\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right) + 8$$
$$= -16\left(\frac{1}{4}\right) + 8 + 8$$
$$= -4 + 8 + 8$$
$$= 12$$
18. (6 points) A farmer wishes to fence off three identical adjoining rectangular pens as in the diagram shown. Determine the maximum total area that can be enclosed by these three pens if he only has 240 linear feet of fencing available. Your answer should include the $x$ and $y$ values which give this maximum area.

\[ A = 3xy \]
\[ A = 3x(60-\frac{3}{2}x) \leq 0 \]
\[ A = 180x - \frac{9}{2}x^2 \]
\[ A' = 180 - 9x \]
\[ A'' = -9 \]

240 = 6x + 4y

So
\[ y = \frac{240 - 6x}{4} \]
\[ y = 60 - \frac{3x}{2} \]

Dimensions
\[ x = 20 \text{ ft} \quad \text{and} \quad y = 30 \text{ ft} \]

Give maximum area of 1800 ft$^2$. 

\[
\text{DIMENSIONS: } \quad x = 20 \text{ ft} \quad \text{and} \quad y = 30 \text{ ft} \\
\text{GIVE MAXIMUM AREA OF } 1800 \text{ ft}^2
\]