Math 122 (Section 5)  Test 2  March 22, 2002

Name _____________________________

RULES FOR THIS TEST:

- Do not borrow another student’s calculator.
- Circle each final answer.
- A correct answer will only be given full credit if enough work is shown to justify that answer.

1. (10 points) Find the equation of the line tangent to the graph of \( f(x) = x^3 - 5x + 6 \) at \( x = 2 \).
2. (40 points) Complete each boxed equation with the appropriate formula for the derivative.

(a) If \( y = 5t^3 - 14\sqrt{t} + \frac{6}{\sqrt{t^2}} \), then

\[
\frac{dy}{dt} =
\]

(b) If \( w = 5e^{(2x^3 - 5x + 7)} \), then

\[
\frac{dw}{dx} =
\]
(c) If \( g(t) = 5te^{-t} \), then

\[ g'(t) = \]

(d) If \( f(x) = \frac{x^8 - 6x^5 + 9}{3x^5} \), then

\[ f'(x) = \]
3. (20 points) Dorothy emptied a bucket of water upon the Wicked Witch of the West who immediately began to melt. If the Scarecrow only had a brain, he would calculate that the witch’s height could now be given by the function \( h(t) = 63(0.91)^t \), where \( t \) is measured in seconds since the water was first thrown upon the witch, and \( h(t) \) is measured in inches.

(a) At time \( t = 11 \) seconds, how tall was the witch and how quickly was she melting? Each answer must be correct to one decimal place.

(b) Given that \( h(3) \approx 47.5 \) and \( h'(3) \approx -4.5 \), use one or more English sentences to clearly explain what all of this means. Assume that your explanation is for someone who knows very little math, so be sure to avoid calculus terms such as derivative, rate of change, function, slope, tangent line, etc.
4. (10 points) Using the axes below, sketch the graph of one function \( f(x) \) which satisfies all of the following properties.

- \( f(0) = 2 \) and \( f(7) = -1 \)
- \( f'(x) < 0 \) for \( x < 5 \), and \( f'(x) > 0 \) for \( x > 5 \)
- \( f''(x) < 0 \) for \( x < 3 \), and \( f''(x) > 0 \) for \( x > 3 \)
5. (20 points) A company in Brazil mines bauxite which it sells to other companies who use the bauxite to produce aluminum. Suppose that \( C(q) \) represents the cost in dollars for this company to mine \( q \) tons of bauxite, and that \( R(q) \) represents the revenue the company will receive when it sells \( q \) tons of bauxite. The graphs of \( C(q) \) and \( R(q) \) are shown below.

![Graph of C(q) and R(q)](image)

(a) What is the exact price that the company charges for each ton of bauxite sold?

(b) At a production level of 7 tons of bauxite, which is greater — the marginal revenue or the marginal cost?

(c) At what production level does marginal revenue equal marginal cost?

(d) Determine the number of tons of bauxite that the company must produce and sell in order to maximize profit. What is the dollar amount of that maximum profit?