1. Which one of the following most clearly states the Fundamental Theorem of Calculus?

   c. Total change in a quantity from \( t = a \) to \( t = b \) equals \( \int_a^b \) (rate of change of that quantity) \( dt \)

2. Which one of the following most clearly states the Fundamental Theorem of Calculus?

   c. \( \int_a^b F'(t) \, dt = F(b) - F(a) \)

3. Evaluate the following definite integrals **without the use of a calculator**. After each problem be sure to also use the built-in integrator on your calculator to check your answers.

   (a) \( \int_1^2 10x^4 \, dx = [2x^5]_1^2 = 2 \cdot 2^5 - 2 \cdot 1^5 = 62 \)

   (b) \( \int_0^2 (5t + 2) \, dt = \left[ \frac{5}{2}t^2 + 2t \right]_0^2 = \left( \frac{5}{2} \cdot 2^2 + 2 \cdot 2 \right) - \left( \frac{5}{2} \cdot 0^2 + 2 \cdot 0 \right) = 14 \)

   (c) \( \int_{-4}^6 3 \, dq = [3q]_{-4}^6 = 3(6) - 3(-4) = 30 \)

   (d) \( \int_{-2}^0 e^w \, dw = [e^w]_{-2}^0 = e^0 - e^{-2} = 1 - \frac{1}{e^2} \approx 0.8647 \)

   (e) \( \int_1^3 \frac{1}{x} \, dx = [\ln x]_1^3 = \ln 3 - \ln 1 = \ln 3 \approx 1.0986 \)

   (f) \( \int_0^3 \left( t^2 - 4t + 3 \right) \, dt = \left[ \frac{1}{3}t^3 - 2t^2 + 3t \right]_0^3 = \left( \frac{1}{3} \cdot 3^3 - 2 \cdot 3^2 + 3 \cdot 3 \right) - \left( \frac{1}{3} \cdot 0^3 - 2 \cdot 0^2 + 3 \cdot 0 \right) = 0 \)

   (g) \( \int_0^2 e^{2q} \, dq = \left[ \frac{1}{2}e^{2q} \right]_0^2 = \frac{1}{2}e^4 - \frac{1}{2} = \frac{1}{2}e^4 - \frac{1}{2} \approx 26.7991 \)

   (h) \( \int_{-1}^1 (w^3 + w) \, dw = \left[ \frac{1}{4}w^4 + \frac{1}{2}w^2 \right]_{-1}^1 = \left( \frac{1}{4}(1)^4 + \frac{1}{2}(1)^2 \right) - \left( \frac{1}{4}(-1)^4 + \frac{1}{2}(-1)^2 \right) = 0 \)

   (i) \( \int_{-1}^3 \frac{6}{x^3} \, dx = \int_{-1}^3 6x^{-3} \, dx = [-3x^{-2}]_1^3 = (-3(3)^{-2}) - (-3(1)^{-2}) = \frac{8}{3} \approx 2.6667 \)

   (j) \( \int_2^3 \frac{1}{4t^2} \, dt = \int_2^3 \frac{1}{4}t^{-2} \, dt = \left[ -\frac{1}{4}t^{-1} \right]_2^3 = \left( -\frac{1}{4}(3)^{-1} \right) - \left( -\frac{1}{4}(2)^{-1} \right) = \frac{1}{24} \approx 0.0417 \)

   (k) \( \int_{-1}^2 e^{-q} \, dq = [-e^{-q}]_{-1}^2 = (-e^{-2}) - (-e^1) = -\frac{1}{e^2} + e \approx 2.5829 \)
(l) \[ \int_1^4 3\sqrt{w} \, dw = \int_1^4 3w^{1/2} \, dw = \left[ 2w^{3/2} \right]_1^4 = (2 \cdot 4^{3/2}) - (2 \cdot 1^{3/2}) = 14 \]

(m) \[ \int_2^4 3x^{-1} \, dx = [3 \ln x]_2^4 = 3 \ln 4 - 3 \ln 2 = 3 \ln 2 \approx 2.0794 \]

(n) \[ \int_0^1 6e^{-3t} \, dt = [-2e^{-3t}]_0^1 = (-2e^{-3}) - (-2e^0) = -\frac{2}{e^3} + 2 \approx 1.9004 \]

(o) \[ \int_9^{25} \frac{4}{\sqrt{q}} \, dq = \int_9^{25} 4q^{-1/2} \, dq = \left[ 8q^{1/2} \right]_9^{25} = 8 \cdot 25^{1/2} - 8 \cdot 9^{1/2} = 16 \]

(p) \[ \int_1^e \frac{2w + 1}{w^2} \, dw = \int_1^e \left( 2w^{-1} + w^{-2} \right) \, dw = \left[ 2 \ln w - w^{-1} \right]_1^e = (2 \ln e - e^{-1}) - (2 \ln 1 - 1^{-1}) = 3 - \frac{1}{e} \approx 2.6321 \]

(q) \[ \int_0^2 xe^{x^2} \, dx = \left[ \frac{1}{2}e^{x^2} \right]_0^2 = \frac{1}{2}e^4 - \frac{1}{2} \approx 26.7991 \]

(r) \[ \int_0^1 \frac{6t^2}{t^3 + 2} \, dt = [2 \ln (t^3 + 2)]_0^1 = 2 \ln (1^3 + 2) - 2 \ln (0^3 + 2) = 2 \ln 3 - 2 \ln 2 = 2 \ln (3/2) \approx 0.8109 \]

(s) \[ \int_0^2 24q(q^2 + 1)^3 \, dq = \left[ 3(q^2 + 1)^4 \right]_0^2 = 3(2^2 + 1)^4 - 3(0^2 + 1)^4 = 1872 \]

(t) \[ \int_0^1 9w^2 e^{w+1} \, dw = [3e^{w+1}]_0^1 = 3e^2 + 3e - 3e^0 - 3e = 3e^2 - 3e \approx 14.0123 \]

(u) \[ \int_0^3 12x\sqrt{x^2 + 16} \, dx = \int_0^3 12x \left( x^2 + 16 \right)^{1/2} \, dx = \left[ 4(x^2 + 16)^{3/2} \right]_0^3 = 4(3^2 + 16)^{3/2} - 4(0^2 + 16)^{3/2} = 244 \]

(v) \[ \int_0^1 3t^2 e^{0.5t^3} \, dt = \left[ 2e^{0.5t^3} \right]_0^1 = 2e^{0.5(1)^3} - 2e^{0.5(0)^3} = 2e^{0.5} - 2 \approx 1.2974 \]

(w) \[ \int_1^3 \frac{3q}{q^2 + 1} \, dq = \left[ \frac{3}{2} \ln (q^2 + 1) \right]_1^3 = \frac{3}{2} \ln (3^2 + 1) - \frac{3}{2} \ln (1^2 + 1) = \frac{3}{2} \ln 10 - \frac{3}{2} \ln 2 = \frac{3}{2} \ln 5 \approx 2.4142 \]

(x) \[ \int_1^2 \ln x \, dx = [x \ln x - x]_1^2 = (2 \ln 2 - 2) - (1 \ln 1 - 1) = 2 \ln 2 - 1 \approx 0.3863 \]
4. Suppose that a town had a population of 5000 people in 1970. If the population was growing at a rate of \( r(t) = 16t \) people per year, where \( t \) represents the number of years since 1970, then what was the population of the town in 1980?

\[
\text{change in population} = \int_0^{10} 16t \, dt = 800 \text{ people}
\]

\[
\text{population} = 5000 \text{ people} + 800 \text{ people} = 5800 \text{ people}
\]

5. The marginal cost of a product, in dollars per item, is \( C'(q) = 3q^2 - 150q + 2100 \). If fixed costs are $50,000 then find the total cost to produce 50 items.

\[
\text{change in cost} = \int_0^{50} (3q^2 - 150q + 2100) \, dq \approx 42500 \text{ dollars}
\]

\[
\text{total cost} = 50000 + 42500 = 92500 \text{ dollars}
\]

6. It is estimated that \( t \) days from now a farmer’s crop will be increasing at the rate of \( 0.3t^2 + 0.6t + 1 \) bushels per day. By how much will the value of the crop increase in the next 5 days if the market price remains fixed at $3 per bushel?

\[
\int_0^5 (0.3t^2 + 0.6t + 1) \, dt \approx 25 \text{ bushels}
\]

At $3 per bushel, the value of the crop will have increased by $75.

7. At 4 AM, the layer of ice on Lake Mendota was 6 inches thick and its thickness was changing at a rate of \( 0.2t \) inches per hour where \( t \) represents the number of hours since 4 AM. How thick was the ice at noon that same day?

\[
\text{change in thickness} = \int_0^8 0.2t \, dt = 6.4 \text{ inches}
\]

\[
\text{thickness at noon} = 6 \text{ inches} + 6.4 \text{ inches} = 12.4 \text{ inches}
\]

8. There are \( 200e^{0.02t} \) deer living on a small island where \( t \) is the number of years since 1980. What is the change in this deer population between 1990 and 2000?

Note that we are not given a rate of change here so we will not integrate the given function. Instead we will directly use the given formula for the number of deer at time \( t \).

\[
\text{change in population} = 200e^{0.02(20)} - 200e^{0.02(10)} \approx 54 \text{ deer}
\]
9. Tom measured the tree in his yard and found that it was 53 inches tall. He then applied a strange new fertilizer which caused the tree to grow at a rate of $35te^{-t}$ inches per hour where $t$ is the number of hours since the application of the fertilizer. How tall was the tree 4 hours after he applied the fertilizer? Your answer should be given in inches and should be accurate to at least 2 places after the decimal point.

\[
\text{change in height} = \int_0^4 35te^{-t} \, dt = 31.79 \text{ inches}
\]

\[
\text{height} = 53 \text{ inches} + 31.79 \text{ inches} = 84.79 \text{ inches}
\]

10. If a population is growing by $5e^{0.03t}$ people per year where $t$ represents the number of years since 1960, then what is the change in population between years 1980 and 1990?

\[
\text{change in population} = \int_{20}^{30} 5e^{0.03t} \, dt \approx 106 \text{ people}
\]

11. A cyclist’s velocity during a 2-hour ride is shown in the graph below. Estimate the distance that she rode?

If we let $v(t)$ be her velocity, then

\[
\text{distance} = \int_0^2 v(t) \, dt \approx 35 \text{ miles}
\]

This answer was obtained by approximating the area under the graph of $v(t)$ between 0 and 2. In particular this region has the equivalent of about 7 little rectangles each of area 5.

12. An object’s velocity in feet per second over a 60 second interval is shown below.
(a) Find all times at which the object is traveling at 60 feet per second.
   24 seconds and 36 seconds

(b) At which time has the object traveled a total distance of 60 feet?
   2 seconds

(c) At the end of the 60 second interval, how fast is the object traveling?
   45 feet per second

(d) What is the total distance the object traveled over this 60 second interval?
   2880 feet since the region under the curve has the equivalent of about 32 little rectangles each of area 90.

13. The graph of $f(x)$ is shown. Given that $\int_{4}^{6} f(x) \, dx = 16$, evaluate the following definite integrals.

(a) $\int_{0}^{2} f(x) \, dx = -16$

(b) $\int_{-2}^{6} f(x) \, dx = 32$

(c) $\int_{3}^{4} f(x) \, dx = 8$

(d) $\int_{6}^{4} f(x) \, dx = -16$

(e) $\int_{3}^{3} f(x) \, dx = 0$

(f) $\int_{0.5}^{3.5} f(x) \, dx = 0$