<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>20</td>
</tr>
<tr>
<td>#2</td>
<td>10</td>
</tr>
<tr>
<td>#3</td>
<td>6</td>
</tr>
<tr>
<td>#4</td>
<td>8</td>
</tr>
<tr>
<td>#5</td>
<td>6</td>
</tr>
<tr>
<td>#6</td>
<td>8</td>
</tr>
<tr>
<td>#7</td>
<td>12</td>
</tr>
<tr>
<td>#8</td>
<td>8</td>
</tr>
<tr>
<td>#9</td>
<td>12</td>
</tr>
<tr>
<td>#10</td>
<td>5</td>
</tr>
<tr>
<td>#11</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Test #1: _____  Test #2: _____  Test #3: _____  Total: _____

If you skip the final exam, your course grade will be: __________________________
Part 1 - no calculators! Show sufficient work to justify each answer.

1. (5 points each) Compute the exact value of each definite integral. Simplify each answer.

   (a) \( \int_{1}^{7} \frac{5}{x} \, dx = \left[ 5 \ln(x) \right]_{1}^{7} = 5 \ln(7) - 5 \ln(1) = 5 \ln(7) \) since \( \ln(1) = 0 \)

   (b) \( \int_{0}^{2} (12x^2) \, dx = \left[ 4x^3 \right]_{0}^{2} = 4(2)^3 - 4(0)^3 = 32 \)

   (c) \( \int_{-3}^{4} dx = \left[ 4x \right]_{-3}^{4} = 4(4) - 4(-3) = 8 + 12 = 20 \)

   (d) \( \int_{1}^{2} 4xe^{x^2-1} \, dx = \left[ 2e^{x^2-1} \right]_{1}^{2} = 2e^3 - 2e \) since \( e^0 = 1 \)

   \( \int \frac{6e^{-x^2}}{2e^{-x^2}} \, dx = \int 3 \, dx = 3x + C \)
Part 2 - calculators are allowed. Show sufficient work to justify each answer.

Name  **SOLUTIONS**

2. (10 points) Coal gas is produced at a gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. The following measurements, made at the start of every other month, show the rate (in tons per month) at which pollutants are escaping in the gas over the course of one year.

<table>
<thead>
<tr>
<th>Time t (months)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate pollutants escape (tons/month)</td>
<td>3.8</td>
<td>5.1</td>
<td>6.8</td>
<td>9.1</td>
<td>12.0</td>
<td>16.0</td>
<td>21.2</td>
</tr>
</tbody>
</table>

(a) Use a right Riemann sum with $\Delta t = 4$ to approximate the total quantity of pollutants that escape over the course of the year.

\[
\begin{align*}
&\left(3.8 \text{ tons/month}\right)(4 \text{ months}) \\
&+ \left(5.1 \text{ tons/month}\right)(4 \text{ months}) \\
&+ \left(6.8 \text{ tons/month}\right)(4 \text{ months}) \\
&= 160 \text{ tons}
\end{align*}
\]

(b) Use a left Riemann sum with $\Delta t = 2$ to approximate the total quantity of pollutants that escape over the course of the year.

\[
\begin{align*}
&\left(3.8 \text{ tons/month}\right)(2) + \left(5.1 \text{ tons/month}\right)(2) + \left(6.8 \text{ tons/month}\right)(2) \\
&+ \left(9.1 \text{ tons/month}\right)(2) + \left(12.0 \text{ tons/month}\right)(2) + \left(16.0 \text{ tons/month}\right)(2) \\
&= 105.6 \text{ tons}
\end{align*}
\]

3. (6 points) Suppose that 450 rabbits are released on Lady Tottington's estate, and that the rabbit population increases by $30e^{0.12t}$ rabbits per month over the next $t$ months. How many rabbits will be on her estate in one year?

\[
\begin{align*}
\text{Change in population} &= \int_{0}^{12} 30e^{0.12t} \, dt \\
&= 805 \\
\text{Population in one year} &= 450 + 805 = 1255 \text{ rabbits}
\end{align*}
\]
4. (8 points) The production of some item has fixed costs of $500 and a marginal cost of $5 + 30e^{-0.02q}$ dollars per item where $q$ represents the number of items.

(a) Find the total cost of producing 300 items.

\[
\text{Total Cost} = \text{Fixed Costs} + \text{Change in Cost} = 500 + \int_{0}^{300} (5 + 30e^{-0.02q}) \, dq \\
\approx 500 + 2996.28 \\
\approx \$3496.28
\]

(b) Determine the increase in cost if production is increased from 300 items to 600 items.

\[
\text{Change in Cost} = \int_{300}^{600} (5 + 30e^{-0.02q}) \, dq \\
\approx \$1503.71
\]

5. (6 points) An oil spill spread out in a circle around the point where an oil tanker was ruptured at 7:00 a.m. Suppose that $t$ minutes later the radius of the oil spill was $80\sqrt{t}$ meters. By what amount did the radius of the oil spill increase between 8:00 a.m. and 8:30 a.m. that morning?

\[
\text{Change in Radius} = \left( \text{Radius at } t = 90 \right) - \left( \text{Radius at } t = 60 \right) \\
= 80\sqrt{90} - 80\sqrt{60} \\
\approx 139.27 \text{ meters}
\]
6. (8 points) The graphs of \( f(x) = x^2 - 10x + 30 \) and \( g(x) = 2x + 3 \) are sketched below and the area between the two curves is shaded in.

\[
\begin{align*}
\text{INTERSECTION} \\
\text{\( x^2 - 10x + 30 = 2x + 3 \)} \\
\text{\( x^2 - 12x + 27 = 0 \)} \\
\text{\( (x - 3)(x - 9) = 0 \)} \\
\text{\( x = 3 \) or \( x = 9 \)}
\end{align*}
\]

(a) Set up, but do not evaluate, the definite integral whose value is equal to the shaded area. Be sure to use proper notation.

\[
\text{AREA} = \int_{3}^{9} \left( (2x+3) - (x^2-10x+30) \right) \, dx
\]

OR

\[
\text{AREA} = \int_{3}^{9} \left( -x^2 + 12x - 27 \right) \, dx
\]

OR

\[
\text{AREA} = \int_{3}^{9} (2x+3) \, dx - \int_{3}^{9} (x^2-10x+30) \, dx
\]

(b) Determine the value of the shaded area.

\[
\text{AREA} = \int_{3}^{9} (-x^2 + 12x - 27) \, dx
\]

\[
= \left[ -\frac{1}{3}x^3 + 6x^2 - 27x \right]_{3}^{9}
\]

\[
= \left( -\frac{1}{3}(9)^3 + 6(9)^2 - 27(9) \right) \\
- \left( -\frac{1}{3}(3)^3 + 6(3)^2 - 27(3) \right)
\]

\[
= 36 \quad \text{(can also built-in)}
\]

\begin{center}
\text{INTEGRATOR}
\end{center}
7. (12 points) An object’s velocity in feet per second over a 40 second interval is shown below.

(a) Find all times at which the object is traveling at 40 feet per second.

\[ \text{TIME} = 15 \text{ sec} \]

(b) At the end of the 40 second interval, how fast is the object traveling?

\[ 48 \text{ ft/sec} \]

(c) What is the total distance the object traveled over this 40 second interval?

\[ \int_0^{40} (\text{velocity}) \, dt = \text{area under graph} \]

\[ = 1600 \text{ ft} \]
8. (8 points) The promoters of a county fair estimate that \( t \) hours after the gates open at 9:00 a.m., visitors will be entering the fair at a rate of \(-3(t + 3)^2(t - 12)\) people per hour. If tickets sell for $6 per person, how much money does the fair bring in between 10:00 a.m. and noon?

\[
\text{# Visitors from 10 a.m. - noon} = \int_{1}^{3} -3(t+3)^2(t-12) \, dt
\]

\[
= 1500 \text{ People}
\]

\[
(1500 \text{ People}) \times (\$6/\text{Person}) = \$9000
\]

9. (12 points) The graph of \( f(x) \) is shown below. Given that \( \int_{2}^{6} f(x) \, dx = 10 \), evaluate the following definite integrals.

(a) \( \int_{4}^{6} f(x) \, dx = 5 \)

(b) \( \int_{0}^{6} f(x) \, dx = -5 + 5 + 5 = 5 \)

(c) \( \int_{1}^{3} f(x) \, dx \approx 0 \)
10. (5 points) Which one of the following most clearly states the Fundamental Theorem of Calculus?

(a) Rate of change of a quantity from $t = a$ to $t = b$ equals $\int_b^a \text{(total change in that quantity)} \ dt$

(b) Rate of change of a quantity from $t = a$ to $t = b$ equals $\int_b^a \text{(rate of change of that quantity)} \ dt$

(c) Total change in a quantity from $t = a$ to $t = b$ equals $\int_a^b \text{(total change in that quantity)} \ dt$

(d) Total change in a quantity from $t = a$ to $t = b$ equals $\int_a^b \text{(rate of change of that quantity)} \ dt$

(e) Rate of change of a quantity from $t = a$ to $t = b$ equals $\int_a^b \text{(total change in that quantity)} \ dt$

(f) Rate of change of a quantity from $t = a$ to $t = b$ equals $\int_a^b \text{(rate of change of that quantity)} \ dt$

(g) Total change in a quantity from $t = a$ to $t = b$ equals $\int_a^b \text{(total change in that quantity)} \ dt$

(h) Total change in a quantity from $t = a$ to $t = b$ equals $\int_a^b \text{(rate of change of that quantity)} \ dt$

11. (5 points) Which one of the following most clearly states the Fundamental Theorem of Calculus?

(a) $\int_a^b F(t) \ dt = F'(b) - F'(a)$

(b) $\int_a^b F'(t) \ dt = F'(b) - F'(a)$

(c) $\int_a^b F(t) \ dt = F(b) - F(a)$

(d) $\int_a^b F'(t) \ dt = F(b) - F(a)$

(e) $\int_a^b F(t) \ dt = F'(a) - F'(b)$

(f) $\int_a^b F'(t) \ dt = F'(a) - F'(b)$

(g) $\int_a^b F(t) \ dt = F(a) - F(b)$

(h) $\int_a^b F'(t) \ dt = F(a) - F(b)$