Do not write below these lines

#1 (16 points)
#2 (6 points)
#3 (6 points)
#4 (6 points)
#5 (6 points)
#6 (6 points)
#7 (6 points)
#8 (6 points)
#9 (6 points)
#10 (16 points)
#11 (10 points)
#12 (5 points)
#13 (5 points)

Total (100 points)

Test #1 _______ Test #2 _______ Test #3 _______ Total _______

If you skip the final exam, your course grade will be ___________________________
Part 1 - no calculators!

1. (4 points each) Compute the exact value of each definite integral. Show sufficient work and simplify each answer.

(a) \[ \int_5^{20} 2 \, dx = \left[2x\right]_5^{20} = 2 \cdot 20 - 2 \cdot 5 = 40 - 10 = 30 \]

(b) \[ \int_1^3 \frac{3}{x} \, dx = \left[3 \ln x\right]_1^3 = 3 \ln 3 - 3 \ln 1 = 3 \ln 3 - 3 \cdot 0 = 3 \ln 3 \]

(c) \[ \int_3^{5} (4x - 2) \, dx = \left[2x^2 - 2x\right]_3^5 = (2 \cdot 5^2 - 2 \cdot 5) - (2 \cdot 3^2 - 2 \cdot 3) = 40 - 12 = 28 \]

(d) \[ \int_0^3 10e^{2x} \, dx = \left[5e^{2x}\right]_0^3 = 5e^{2 \cdot 3} - 5e^0 = 5e^6 - 5 \]
2. (6 points) A car is traveling at 80 feet per second when the driver sees a deer in the road 350 feet ahead and immediately steps on the brakes. The deer freezes and does not move from his spot in the road. I've recorded the driver's speed (in ft/sec) every two seconds starting at the time that he first stepped on the brakes and going until the time that the car finally came to a stop. From the moment the driver steps on the brakes, determine both an underestimate and overestimate of the distance the car travels before coming to a stop. You may assume that the car is always slowing down until it comes to a stop. Does the car hit the deer?

<table>
<thead>
<tr>
<th>time (sec)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>car's speed (ft/sec)</td>
<td>80</td>
<td>50</td>
<td>25</td>
<td>15</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
\text{RIGHT SUM} \quad 50.2 + 25.2 + 15.2 + 0.2 = 180 \text{ ft}
\]

\[
\text{UNDERESTIMATE}
\]

\[
\text{LEFT SUM} \quad 80.2 + 50.2 + 25.2 + 15.2 = 1340 \text{ ft}
\]

\[
180 \text{ ft} \leq \text{stopping distance } \leq 1340 \text{ ft}
\]

\[
\text{THE CAR DOES NOT HIT THE DEER}
\]

3. (6 points) A town currently has a population of 4000 people but its population is expected to grow by \(0.5e^t\) people per year over the next 15 years. What is the town's population expected to be 15 years from now?

\[
4000 + \int_0^{15} 0.5e^t \, dt
\]

\[= 4000 + 962.5 \]

\[= 4562.5 \text{ people}
\]

\[\approx 4562 \text{ or } 4563 \text{ people}
\]
4. (6 points) A town has a population of $3000 + 20t$ people where $t$ represents the number of years since 1960. What is the change in population for this town between years 1980 and 1990?

$$P(t) = 3000 + 20t$$

$$\Delta P = P(30) - P(20)$$

$$= 3600 - 3400$$

$$= 200 \text{ PEOPLE}$$

5. (6 points) The marginal cost for the production of some item is given by the formula $\frac{35}{\sqrt{q + 1}}$ where $q$ represents the number of items. By what amount does the cost increase if production is increased from 500 items to 750 items?

$$\int_{500}^{750} \frac{35}{\sqrt{q + 1}} \, dq = \$351.49$$
6. (6 points) An oil spill spread out in a circle around the point where an oil tanker was ruptured. At 5pm the radius of the oil spill was 100 feet. From that point on the radius was increasing by \( \ln(t+1) \) feet per minute where \( t \) represents the number of minutes past 5pm. What was the radius of the oil spill at 5:30pm?

\[
100 + \int_0^{30} \ln(t+1) \, dt \\
\approx 100 + 76.5 \\
\approx 176.5 \text{ ft}
\]

7. (6 points) Sketch the graph of \( y = 16 - x^2 \) on the interval \([0, 5]\) and shade in the area between the \( x \)-axis and this graph. Compute the value of this shaded area.

\[
\text{AREA} = \int_0^4 (16 - x^2) \, dx + \int_4^5 (16 - 5^2) \, dx \\
= 42.6 + 1 - 4.31 \\
= 42.6 + 4.31 \\
= 47
\]
8. (6 points) The graphs of \( f(x) = 4x^2 - 5 \) and \( g(x) = 4x + 3 \) are sketched below and the area between the two curves is shaded in.

\[
\begin{align*}
4x^2 - 5 &= 4x + 3 \\
4x^2 - 4x - 8 &= 0 \\
4(x^2 - x - 2) &= 0 \\
4(x - 2)(x + 1) &= 0 \\
x &= 2 \text{ or } x = -1
\end{align*}
\]

(a) Set up, but do not evaluate, the definite integral whose value is equal to the shaded area. Be sure to use proper notation.

\[
\text{AREA} = \int_{-1}^{2} ((4x + 3) - (4x^2 - 5)) \, dx
\]

(b) Determine the value of the shaded area.

\[
\text{AREA} = 18
\]

9. (6 points) It is estimated that \( t \) days from now a farmer's crop will be increasing at the rate of \( 2t + 5 \) bushels per day. By how much will the value of the crop increase in the next 6 days if the market price remains fixed at $4 per bushel?

\[
\begin{align*}
\text{INCREASE IN \#BUSHELS} &= \int_{0}^{6} (2t + 5) \, dt = 66 \\
\text{INCREASE IN VALUE} &= (66 \text{ BUSHELS}) \times ($4/\text{BUSHEL}) \\
&= $264
\end{align*}
\]
10. (16 points) An object's velocity in feet per second over a 60 second interval is shown below.

(a) Find all times at which the object is traveling at 60 feet per second.

\[30 \text{ sec and } 42 \text{ sec}\]

(b) At which time has the object traveled a total distance of 60 feet?

\[4 \text{ sec}\]

(c) At the end of the 60 second interval, how fast is the object traveling?

\[30 \text{ ft/sec}\]

(d) What is the total distance the object traveled over this 60 second interval?

\[27.90 = 2430 \text{ ft}\]
11. (10 points) The graph of $f(x)$ is shown below. Given that $\int_{4}^{6} f(x) \, dx = 8$, evaluate the following definite integrals.

(a) $\int_{0}^{4} f(x) \, dx = -8$

(b) $\int_{3}^{3} f(x) \, dx = 0$

(c) $\int_{0}^{2} f(x) \, dx = -8$

(d) $\int_{-2}^{6} f(x) \, dx = 16$

(e) $\int_{3}^{4} f(x) \, dx = 4$
12. (5 points) Which one of the following most clearly states the Fundamental Theorem of Calculus?

(a) Total change in a quantity from \( t = a \) to \( t = b \) equals \( \int_{a}^{b} \) (rate of change of that quantity) \( dt \)

(b) Total change in a quantity from \( t = a \) to \( t = b \) equals \( \int_{a}^{b} \) (rate of change of that quantity) \( dt \)

(c) Total change in a quantity from \( t = a \) to \( t = b \) equals \( \int_{a}^{b} \) (total change in that quantity) \( dt \)

(d) Total change in a quantity from \( t = a \) to \( t = b \) equals \( \int_{a}^{b} \) (total change in that quantity) \( dt \)

(e) Rate of change of a quantity from \( t = a \) to \( t = b \) equals \( \int_{a}^{b} \) (rate of change of that quantity) \( dt \)

(f) Rate of change of a quantity from \( t = a \) to \( t = b \) equals \( \int_{a}^{b} \) (rate of change of that quantity) \( dt \)

(g) Rate of change of a quantity from \( t = a \) to \( t = b \) equals \( \int_{a}^{b} \) (total change in that quantity) \( dt \)

(h) Rate of change of a quantity from \( t = a \) to \( t = b \) equals \( \int_{a}^{b} \) (total change in that quantity) \( dt \)

13. (5 points) Which one of the following most clearly states the Fundamental Theorem of Calculus?

(a) \( \int_{a}^{b} F(t) \, dt = F'(a) - F'(b) \)

(b) \( \int_{a}^{b} F(t) \, dt = F'(b) - F'(a) \)

(c) \( \int_{a}^{b} F'(t) \, dt = F'(a) - F'(b) \)

(d) \( \int_{a}^{b} F'(t) \, dt = F'(b) - F'(a) \)

(e) \( \int_{a}^{b} F(t) \, dt = F(a) - F(b) \)

(f) \( \int_{a}^{b} F(t) \, dt = F(b) - F(a) \)

(g) \( \int_{a}^{b} F'(t) \, dt = F(a) - F(b) \)

(h) \( \int_{a}^{b} F'(t) \, dt = F(b) - F(a) \)