Name ________________

Seat # ___________________

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For all questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student's calculator during the test.
- Show your Student ID when you turn in your test.

Do not write below this line

#1 (10 points) __________________

#2 (10 points) __________________

#3 (10 points) __________________

#4 (10 points) __________________

#5 (10 points) __________________

#6 (10 points) __________________

#7 (10 points) __________________

#8 (10 points) __________________

#9 (10 points) __________________

#10 (10 points) __________________

Total (100 points) __________________
1. (10 points) In Scotland the number of sea trout has been declining very rapidly. This is believed to be due to parasites from nearby farmed fish populations. The number of sea trout caught in 1985 by anglers staying at the Loch Maree Hotel was 1100. This yearly catch then decreased at a continuous rate of 58% per year. What was the number of sea trout caught by anglers staying at this hotel 4 years later?

\[ A(t) = 1100 e^{-0.58t} \]

\[ A(4) = 1100 e^{-0.58(4)} \approx 108 \text{ sea trout} \]

2. (10 points) Twenty years ago, Tom inherited some money which he promptly invested in an account earning interest at 8.5% compounded annually. Today the amount of money in this account is $15,000. Assuming he never made any withdrawals or further deposits, what was the amount of his inheritance?

\[ A = A_0 (1.085)^t \]

\[ 15000 = A_0 (1.085)^{20} \]

\[ A_0 = \frac{15000}{(1.085)^{20}} \]

\[ A_0 \approx 2934.25 \]
3. (10 points) A company could make 150 dolls at a total cost of $966, and it could make 200 dolls at a total cost of $1136. When the company sells these dolls, they charge $5 per doll.

(a) Determine a formula for a function $R(q)$ which represents the revenue generated when the company sells $q$ dolls.

$$R(q) = 5q$$

(b) Determine a formula for a linear function $C(q)$ which represents the total cost for the company to manufacture $q$ dolls.

\[
\begin{align*}
\text{Slope } &= \frac{1136 - 966}{200 - 150} \\
&= 3.4
\end{align*}
\]

\[
C(q) - 966 = 3.4(q - 150)
\]

$$C(q) = 3.4q + 456$$

(c) Determine the fixed costs to the company.

$$C(0) = 456$$

(d) If the company produces and sells 250 dolls, do they earn a profit or suffer a loss? What is the dollar amount of that profit (or loss)?

\[
\begin{align*}
\text{Profit} &= R(250) - C(250) \\
&= 1250 - 1306 \\
&= -56
\end{align*}
\]

(e) What is the exact number of dolls which the company must manufacture and sell in order to break even? (i.e., to have a profit of $0)?

\[
\begin{align*}
5q &= 3.4q + 456 \\
q &= 285 \text{ Dolls}
\end{align*}
\]
4. (10 points) Kathryn started reading the newest Harry Potter book when she purchased it at midnight. Let \( P(t) \) represent the total number of pages she read in the \( t \) hours since midnight. Avoiding mathematical terms such as derivative, function, rate of change, etc., write one or two complete English sentences which together describe what is meant by the two equations below.

\[
P'(2) = 100 \quad \text{and} \quad P'(2) = 40
\]

AT 2 AM KATHRYN HAD FINISHED 100 PAGES AND AT THAT TIME WAS READING 40 PAGES PER HOUR.

5. (10 points) Solve for \( x \) in each of the following equations. To receive any credit, you must show the work done by hand to obtain your answer and leave your answer in exact form.

(a) \( 10e^{2x} + 8 = 98 \)

\[
\begin{align*}
10e^{2x} &= 90 \\
e^{2x} &= 9 \\
\ln (e^{2x}) &= \ln 9 \\
2x &= \ln 9 \\
x &= \frac{\ln 9}{2}
\end{align*}
\]

(b) \( \ln (x^7) - \ln (x^3) = 12 \)

\[
\begin{align*}
7\ln x - 3\ln x &= 12 \\
4\ln x &= 12 \\
\ln x &= 3 \\
x &= e^3
\end{align*}
\]
6. (10 points) Suppose that the population of a city $t$ years from today is modeled by the function

$$P(t) = 3500(1.06)^t$$

(a) On average, how quickly is the population predicted to increase over the next 7 years?

$$\frac{\Delta P}{\Delta t} = \frac{P(7) - P(0)}{7 - 0}$$

$$= \frac{3500(1.06)^7 - 3500}{7}$$

$$\approx 251.8 \text{ people/year}$$

(b) How quickly is the population predicted to be increasing precisely 7 years from today?

$$P'(7) = \frac{P(7.01) - P(7)}{7.01 - 7}$$

$$= \frac{3500(1.06)^{7.01} - 3500(1.06)^7}{0.01}$$

$$\approx 306.7 \text{ people/year}$$
7. (10 points) A table of values is given for three different functions.

- Circle **linear** if the function could be linear, circle **exponential** if the function could be exponential, and circle **neither** if it is impossible for the function to be either linear or exponential.

- Using correct variables and function names, find a formula for at least 2 of the functions.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$f(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
</tr>
<tr>
<td>4</td>
<td>3.6</td>
</tr>
<tr>
<td>6</td>
<td>10.8</td>
</tr>
</tbody>
</table>

(a) \[ f(t) = 0.4(3)^{t/2} \]

or \[ f(t) = 0.4 (1.732)^t \]

or \[ f(t) = 0.4 e^{0.549t} \]

linear, exponential, neither

<table>
<thead>
<tr>
<th>$t$</th>
<th>$g(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
</tr>
</tbody>
</table>

(b) \[ g(t) = \left(\frac{t}{3} + 1\right)^2 \]

linear, exponential, neither

<table>
<thead>
<tr>
<th>$t$</th>
<th>$h(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.2</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
</tr>
<tr>
<td>4</td>
<td>2.8</td>
</tr>
<tr>
<td>6</td>
<td>2.1</td>
</tr>
</tbody>
</table>

(c) \[ h(t) = -0.35t + 4.2 \]

linear, exponential, neither
8. (10 points) Accurate to two places after the decimal point, find the $x$-values of all intersection points on the graphs of $y = 6 + e^{-x}$ and $y = 18 - 0.1x^2$.

9. (10 points) Suppose that $f(x) = \ln (x^{10} + 10^x)$. Approximate the value of $f'(5)$. Your answer should be rounded off correctly to one place after the decimal point.

$f'(5) \approx 2.0$
10. (10 points) Strontium-90, a waste product from nuclear reactors, has a half-life of 28 years. Suppose that a nuclear accident occurs and releases 250 grams of strontium-90 into the atmosphere. How long will it take for the strontium-90 to decay to a level of 25 grams? Round off your answer correctly to one place after the decimal point.

\[ A = 250 e^{kt} \]

\[ 125 = 250 e^{k \cdot 28} \]

\[ 0.5 = e^{28k} \]

\[ \ln(0.5) = 28k \]

\[ k = \frac{\ln(0.5)}{28} \]

\[ k \approx -0.02476 \]

\[ A = 250 e^{-0.02476t} \]

\[ 25 = 250 e^{-0.02476t} \]

\[ 0.1 = e^{-0.02476t} \]

\[ \ln(0.1) = -0.02476t \]

\[ t = \frac{\ln(0.1)}{-0.02476} \]

\[ t \approx 93.0 \text{ years} \]