Chapter 4
Sec. 4.1 (Num. 3) Minimization Problems

Solve
\[ f_1(x) = 0, \ldots, f_m(x) = 0, \quad x \in \mathbb{R}^n \quad (x) \]

Set \[ p(x) = (f_1(x))^2 + \cdots + (f_m(x))^2 \]

So \( p(x) \geq 0 \) and \( p(x) = 0 \) only if \( x \) satisfies \((x)\). So solutions also satisfy

\[
\minimize_{x \in \mathbb{R}^n} p(x).
\]

If the minimum is zero, we have solutions. What if it isn't? Then no solution. But we can use \( p(x) \) to measure the degree to which the problem cannot be satisfied.

Data Fitting

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>y_1</td>
</tr>
<tr>
<td>1.1</td>
<td>\text{see graph} y_2</td>
</tr>
<tr>
<td>1.2</td>
<td>\text{...}</td>
</tr>
<tr>
<td>2.0</td>
<td>y_{n}</td>
</tr>
</tbody>
</table>

Can we fit \( y = mx + b \) to the data? What does this mean?

\[
y_1 = m \cdot 1 + b
\]
\[
y_2 = m \cdot 1.1 + b
\]
\[
y_3 = m \cdot 1.2 + b \quad \cdots \quad y_n = m \cdot 2 + b \quad \text{No solution.}
\]

But closer makes more...