Math 230  
Mastery Test III  
March 17, 2005

Name: ____________________________

Instructor: Jeremy Tyson / Micah James

Directions:

• You may not use any books or notes.

• You may not use any kind of computing device (calculator, computer, etc.).

• Do all problems in the space provided.

• Show **ALL** work. Make sure that your work is legible and neatly ordered. Credit will **NOT** be given if you only give the final answer.

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Score:

1. _______ / 20

2. _______ / 15

3. _______ / 15

4. _______ / 15

5. _______ / 15

6. _______ / 20

TOTAL. _______ / 100
1. (20 points) Let \( f(x) = \ln(1 + x) \) and let \( I = \int_{1}^{5} f(x) \, dx \).

(a) Explain why \(|f'(x)| \leq \frac{1}{2}\) and \(|f''(x)| \leq \frac{1}{4}\) for all \( x \) in \([1, 5]\).

(b) Use part (a) to find error bounds for \( R_{10} \) and \( M_{5} \).

2. (15 points) Consider the IVP \( y' = 1 + xy \) with \( y(0) = 1 \).

(a) Use Euler’s method with one step of size 4 to estimate \( y(4) \).

(b) Use Euler’s method with four steps of size 1 to estimate \( y(4) \).
3. (15 points) Find the arc length of the graph of the function \( y = \frac{1}{3}x^{3/2} \) between \( x = 4 \) and \( x = 8 \).

4. (15 points) Sketch the region bounded by the graphs of \( y = x^3 \), \( y = 16 - x^3 \), \( x = 0 \) and \( x = 2 \). Find the volume of the solid obtained by revolving this region about the \( x \)-axis.
5. Find the solution to the IVP \( y' = xy^2 \), \( y(0) = 1 \). Give your answer in the form \( y = f(x) \).
6. (20 points) (a) Find $\int xe^{3x} \, dx$. Check your answer by differentiation.

(b) Find $\int e^x \sin x \, dx$. Check your answer by differentiation.