1. Find all local extrema of the function 

\[ f(x) = \frac{x}{x^2 + 4} \]

The derivative of \( f \) is 

\[
 f'(x) = \frac{(x^2 + 4)(x)' - x(x^2 + 4)'}{(x^2 + 4)^2} \\
= \frac{x^2 + 4 - 2x^2}{(x^2 + 4)^2} \\
= \frac{4 - x^2}{(x^2 + 4)^2}
\]

Solution:

The expression for \( f' \) shows that \( f'(x) \) exists everywhere and so the critical numbers and hence the local extrema occur where \( f'(x) = 0 \). From the derivative this happens when \( 4 - x^2 = 0 \), that is, at \( x = -2 \) and \( x = 2 \).

To determine which of these are local maxs or local mins, we need to do either of the following calculations:

**First Derivative Test**: From the expression for \( f' \) we see that \( f' \) is increasing on \((-2, 2)\) and decreasing on \((-\infty, -2)\) and \((2, \infty)\). This implies that \( f \) is increasing on \((-2, 2)\) and decreasing on \((-\infty, -2)\) and \((2, \infty)\). By the first derivative test, \( x = -2 \) is a local min and \( x = 2 \) is a local max.

**Second Derivative Test**: The second derivative of \( f \) is 

\[
 f''(x) = \frac{(x^2 + 4)^2(4 - x^2)' - (4 - x^2)((x^2 + 4)^2)'}{(x^2 + 4)^4} \\
= \frac{(x^2 + 4)^2(-2x) - (4 - x^2)2(x^2 + 4)(2x)}{(x^2 + 4)^4} \\
= \frac{(x^2 + 4)(-2x) - (4 - x^2)2(2x)}{(x^2 + 4)^3} \\
= \frac{-2x^3 - 8x - 16x + 4x^3}{(x^2 + 4)^3} \\
= \frac{2x^3 - 24x}{(x^2 + 4)^3} \\
= \frac{2x(x^2 - 12)}{(x^2 + 4)^3}
\]

Thus \( f''(2) = 2 \cdot 2 \cdot (-8)/(8)^3 < 0 \) and so \( x = 2 \) is a local max. Similarly \( f''(-2) = 2 \cdot (-2) \cdot (-8)/(8)^3 > 0 \) and so \( x = -2 \) is a local min.

2. Determine the intervals where the graph of 

\[ g(x) = x^3 - 9x^2 + 2x - 1 \]
is concave up and concave down. Express your answer in interval notation. Is there an inflection point?

Solution: The derivative of $g$ is

$$g'(x) = 3x^2 - 18x + 2$$

Therefore the second derivative of $g$ is

$$g''(x) = 6x - 18 = 6(x - 3)$$

We see that $g''(x) > 0$ on $(3, \infty)$ and $g''(x) < 0$ on $(-\infty, 3)$. This implies that $g$ is concave up on $(3, \infty)$ and concave down on $(-\infty, 3)$. This also shows that $(3, g(3))$ is an inflection point.