Math 220 Quiz # 3 Solutions

1. Compute the derivative of

\[ f(x) = (x^2 + 7x + 4) \tan x \]

Solution:

\[
 f'(x) = (x^2 + 7x + 4)' \tan x + (x^2 + 7x + 4)(\tan x)' \\
 = (2x + 7) \tan x + (x^2 + 7x + 4) \sec^2 x
\]

2. Compute the derivative of

\[ g(x) = \frac{7x^2 + 5}{x^2 + 6} \]

Solution:

\[
 g'(x) = \frac{(x^2 + 6)(7x^2 + 5)' - (x^2 + 6)'(7x^2 + 5)}{(x^2 + 6)^2} \\
 = \frac{14x(x^2 + 6) - 2x(7x^2 + 5)}{(x^2 + 6)^2}
\]

3. Find the equation of the tangent line for

\[ y = f(x) = \sqrt{x^2 + 99} \]

at \( x = 1 \).

Solution:

When \( x = 1 \), we have \( y = f(1) = \sqrt{1^2 + 99} = \sqrt{100} = 10 \). Thus the tangent passes through \((1, 10)\). To find the slope of the tangent, compute the derivative:

\[
 f'(x) = \frac{1}{2}(x^2 + 99)^{-1/2}(x^2 + 99)' \\
 = \frac{1}{2}(x^2 + 99)^{-1/2}(2x) \quad \text{Chain Rule} \\
 = \frac{x}{\sqrt{x^2 + 99}}
\]

Thus \( m = f'(1) = 1/10 \). Therefore the equation of the tangent line is

\[ y - 10 = \frac{1}{10}(x - 1) \]