1. (10 points)
Find an expression for the derivative of the following function. You do not need to simplify your answer once all derivatives have been computed.

\[ f(x) = (3x^3 + 4x) \tan^{-1} x \]

\[ f'(x) = (3x^3 + 4x) \tan^{-1} x + (3x^3 + 4x) \left( \frac{1}{1 + x^2} \right) \]

\[ = \left( 9x^2 + 4 \right) \tan^{-1} x + \frac{3x^3 + 4x}{1 + x^2} \]

Line one is not needed if line two makes it clear that line one was used correctly, though implicitly.

2. (10 points)
(a) Find the derivative of

\[ g(t) = e^{3-12t} \]

(b) For which POSITIVE values of \( t \) is \( g'(t) = 0? \)
1. (10 points)
Find an expression for the derivative of the following function. You do not need to simplify your answer once all derivatives have been computed.

\[ f(x) = (3x^3 + 4x) \tan^{-1} x \]

2. (10 points)
(a) Find the derivative of

\[ g(t) = e^{t^3 - 12t} \]

(b) For which POSITIVE values of \( t \) is \( g'(t) = 0 \)?

\[ \text{By the Chain Rule,} \quad g'(t) = e^{t^3 - 12t} \cdot (t^3 - 12t)' \]

\[ = (3t^2 - 12) e^{t^3 - 12t} \]

Line 1 isn’t needed if line 2 is done correctly.

b) Since \( e^{\text{anything}} > 0 \), the only way to have \( g'(t) = 0 \) is to have \( 3t^2 - 12 = 0 \)

\[ \Rightarrow t^2 = 4 \text{ and so } t = 2 \]

\[ \Rightarrow -1 \text{ if negative used} \]
3. (10 points)
Find an expression for the derivative of the following function. You do not need to simplify your answer once all derivatives have been computed.

\[
f(x) = \frac{17x + \log_2 x}{x^{17}}
\]

\[
f'(x) = \frac{(17x + \log_2 x)' x^{17} - (17x + \log_2 x)(x^{17})'}{(x^{17})^2}
\]

\[
= \frac{(17 + \frac{1}{x \log_2 2}) x^{17} - 17x^6 (17x + \log_2 x)}{x^{34}}
\]

Line 1 is not needed if line 2 makes it clear that line 1 was used correctly, though implicitly.

4. (10 points)
Using ONLY the definition of a derivative as a limit, find the derivative function \( f'(x) \) for the function

\[
f(x) = \sqrt{x}
\]
3. (10 points)
Find an expression for the derivative of the following function. You do not need to simplify your answer once all derivatives have been computed.

\[ f(x) = \frac{17x + \log_2 x}{x^{17}} \]

4. (10 points)
Using ONLY the definition of a derivative as a limit, find the derivative function \( f'(x) \) for the function \( f(x) = \sqrt{x} \)

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}
= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}
= \lim_{h \to 0} \frac{x+h - x}{h(\sqrt{x+h} + \sqrt{x})}
= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}
= \frac{1}{2\sqrt{x}}
\]
5. (10 points)
Find the equation of the tangent line to the following curve at \( x = 0 \)

\[ y = (5x + 1)^8 \]

When \( x = 0 \), \( y = (5 \cdot 0 + 1)^8 = 1 \), so our point is \((0, 1)\).

Also \( y' = 8(5x+1)^7 \cdot 5 \) \( \text{by Chain Rule} \) \(3\text{pts}\)

Thus \( y'(0) = 40(5 \cdot 0 + 1)^7 = 40 \). \(\text{2 pts}\)

Therefore the slope of the tangent is

\[ y - 1 = 40x \]

\[ \text{Give partial credit where it makes sense.} \]

6. (10 points)
Find the value of the slope of the tangent at the point \((1, 2)\) to the curve that satisfies the equation

\[ x^3 + xy + y^3 = 11 \]

\[ y'(x) = 7(3x+1)^6 \cdot 3 \]

\[ y = 21x + 1 \]
5. (10 points)
Find the equation of the tangent line to the following curve at \( x = 0 \)

\[ y = (5x + 1)^8 \]

6. (10 points)
Find the value of the slope of the tangent at the point \((1, 2)\) to the curve that satisfies the equation

\[ x^3 + xy + y^3 = 11 \]

\[
\frac{d}{dx}:
\]

\[
x^3 + xy(x) + (y(x))^3 = 1
\]

\[
3x^2 + (x)\frac{dy}{dx} + x y'(x) + 3(y(x))^2y'(x) = 0
\]

\[
3x^2 + y(x) + x y'(x) + 3 y^2(x) y' = 0
\]

\[
\text{Now we can set } x=1 \text{ and } y=2 \text{ to get}
\]

\[
3 \cdot 1^2 + 2 + 1 \cdot y'(1) + 3 (2)^2 y'(1) = 0
\]

\[
13 y'(1) = -5
\]

\[
y'(1) = -\frac{5}{13}
\]

\[
\frac{dy}{dx} = \frac{-y - 3x^3}{x + 3y^2}
\]

\[ -\frac{13}{5} \]
7. (10 points)
Find the derivative of the function

\[ h(x) = x \cos x \]

Set \( y = x \cos x \)
Then \( \ln y = \ln x \cos x = \cos x \ln x \)
So \( \frac{1}{y} y' = (\cos x)' \ln x + \cos x (\ln x)' \)
\[ \frac{1}{y} y' = -\sin x \ln x + \frac{\cos x}{x} \]
So \( y' = (-\sin x \ln x + \frac{\cos x}{x})y \)

8. (10 points)
(a) Given a function \( f \) that is continuous on the closed interval \([a, b]\) and differentiable on the open interval \((a, b)\), what property of \( f \) does the Mean Value Theorem give us? Illustrate this property with a graph that suggests why it should be true.

(b) Given the function \( f(x) = x^3 \) on the interval \([0, 6]\), find a point \( c \) in this interval for which the conclusion of the Mean Value Theorem is true.
7. (10 points)
Find the derivative of the function

\[ h(x) = x^{\cos x} \]

8. (10 points)

(a) Given a function \( f \) that is continuous on the closed interval \([a, b]\) and differentiable on the open interval \((a, b)\), what property of \( f \) does the Mean Value Theorem give us? Illustrate this property with a graph that suggests why it should be true.

(b) Given the function \( f(x) = x^3 \) on the interval \([0, 6]\), find a point \( c \) in this interval for which the conclusion of the Mean Value Theorem is true.

\[ a) \text{ There is a } c \text{ between } a \text{ and } b \text{ at which } \\
\frac{f(b)-f(a)}{b-a} = \frac{b^3-a^3}{b-a} = b^2 = 36 \quad \{2 \text{ pt}\} \]

\[ f'(x) = 3x^2. \text{ So we need to solve } 3c^2 = 36, \text{ i.e. } c^2 = 12 \quad \{2 \text{ pt}\} \]

\[ (\text{not } -\sqrt{12} \text{ since that is not in } (0, 6)) \]

\[ -1 \text{ is included.} \]