Lecture 18  Sec 16.1

Vector Fields (Have e-text open on screen)

Defn: Let $D$ be a region in $\mathbb{R}^2$. Recall a function $F : D \to \mathbb{R}^2$, a vector field on $D$. $F(x,y) = \langle P(x,y), Q(x,y) \rangle$.

(D could be in $\mathbb{R}^3$ and $F : D \to \mathbb{R}^3$)

See e-text for examples and discussion.

Ex. $F(x,y) = -yi + xj$ (See table and graph in e-text)

Looks like $F$ is $\perp$ to position vector!

$p(x,y) = xi + yj$ (another vector field)

$p \cdot F = -xy + xy = 0$

arrows as long as distance to $0$.

Now look at Figures 6, 7, 8 in e-text and discuss

Then look at Figures 10, 11, 12 and discuss

Ex. Newton's Law of Gravitation

$$F(x,y,z) = -\frac{mMg}{l(x,y,z)^2}$$

Inverse Square Law

Ex. Electric Field

Electric field produced by $Q$ is $E(x,y,z) = \frac{1}{q}F = \frac{Qe}{r^2}$
Ex: Gradient Field

\[ \nabla f(x, y) = f_x(x, y)i + f_y(x, y)j \]

How is this related to a contour plot of \( f \)?

Invite students to come up and draw \( \nabla f \) at selected points!

Define a vector field \( F \) is conservative if \( F = \nabla f \) for some scalar function \( f \).

Begin discussions of line integrals.