Recall: \[ \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \]
or \[ \lim_{x \to a} \frac{f(x) - (f(a) + f(a)(x - a))}{x - a} = 0 \]

So differentiable \( \iff \) \( L(x) \) is a good approx of \( f(x) \) near \( x = a \).

For \( f(x,y) \) we proceed in the same way:

\[ f(x,y) \text{ is differentiable at } (a,b) \text{ if } \]
\[ \lim_{(x,y) \to (a,b)} \frac{f(x,y) - (f(a,b) + f_x(a,b)(x - a) + f_y(a,b)(y - b))}{\sqrt{(x - a)^2 + (y - b)^2}} = 0 \]

Recall how tricky this can be!

**Theorem:** If \( f_x \) and \( f_y \) exist and are continuous at \((a,b)\), then \( f \) is differentiable at \((a,b)\).

**Remark:** Differentials

\[ \Delta y = f(a+dx) - f(a) \] change in height of function
\[ dy = f(a) \, dx \] change in height of tangent line

Similarly for \( \Delta z = f(x,y) \) near \((a,b)\)

\[ \Delta z = f(a+dx, b+dy) - f(a,b) \] (rise in fin)
\[ dz = f_x(a,b) \, dx + f_y(a,b) \, dy \] (rise in tgt plane)

differentials \( \Delta z = 0 \) at \((a,b)\) changes to \((a+dx, b+dy)\)
Recall from calc I: \( y = f(x(t)) \)
\[
\frac{dy}{dt} = f'(x(t)) \frac{dx}{dt}
\] Chain Rule

viewed in one way this comes from \( \frac{dy}{dt} = f'(x) \frac{dx}{dt} \) by dividing by \( dt \). The same idea gives us the chain rule for functions of several variables:

\[
dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy
\]

leads to

\[
z(t) = z(x(t), y(t)) \Rightarrow \frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}
\]

Ex. \( z = xy + 3xy' \) where \( x = \sin 2t \), \( y = \cosh t \)
Find \( \frac{dz}{dt} \) when \( t = 0 \)  Ans: 6

Now think about \( z(s,t) = z(x(s,t), y(s,t)) \)

\[
\frac{\partial z}{\partial s} = \ldots > \frac{\partial z}{\partial t} = \ldots
\]

Ex. \( u = x^4 y + y^2 z^2 \) where \( x = rs \cos t \), \( y = rs \cos t \), \( z = rs \sin t \)
Find \( \frac{\partial u}{\partial s} \) when \( r = 2, s = 1, t = 0 \)  Ans: 192