So far we have seen surfaces in 3D that are spheres or planes. Here we want to talk about 3D versions of the conic sections (ellipses, parabolas, hyperbolas) called quadrics

\[ Ax^2 + By^2 + Cz^2 + J = 0 \] or \[ Ax^2 + By^2 + Iz = 0 \]

(by translations and rotations and quadratic in \( x, y, z \) becomes one of these two).

Traces are the curves of intersection with plane parallel to the coordinate planes.

**Cylinders.** A surface consisting of all lines parallel to a given line and passing through a plane curve.

**Ex** \( z = x^2 \), \( y \)-orbit

\[ \text{trace for } y = 0 \]

\[ \text{trace for } y = 1 \]

Ex \( x^2 + y^2 = 1 \), \( z \)-orbit:

Ex \( y^2 + z^2 = 1 \), \( x \)-orbit (lines parallel to \( x \)-axis through a circle of radius 1 in \( yz \)-plane)
Ex \( x^2 + \frac{y^2}{q} + \frac{z^2}{4} = 1 \) \( \Rightarrow \) ellipsoid.

Traces for \( z = k \):
\[ x^2 + \frac{y^2}{q} = 1 - \frac{k^2}{4} \] ellipse

Traces for \( x = k \):
\[ \frac{y^2}{q} + \frac{z^2}{4} = 1 - k^2 \] "

Traces for \( y = k \):
\[ x^2 + \frac{z^2}{4} = 1 - \frac{k^2}{q} \] "

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]

\[ \text{Ex} \quad z = 4x^2 + y^2 \]

Trace for \( z = k \) \( \Rightarrow \) \( 4x^2 + y^2 = k \) ellipses

Trace for \( x = k \) \( \Rightarrow \) \( z - 4k^2 = y^2 \) parabola in \( yz \) plane

Trace for \( y = k \) \( \Rightarrow \) \( z - k^2 = 4x^2 \) \( xz \) plane

\[ \text{Ex} \quad z = y^2 - x^2 \]

\[ x = k \Rightarrow z + k^2 = y^2 \]

\[ z = k \]

\[ k = 0 \]

\[ k = \pm 1 \]

\[ y \]

\[ k = \pm 2 \]
Now review the standard conics on pg 830 and identify features of each!

Sec 14.1

Here we begin our journey into functions of several variables. We begin with two variables $x$ and $y$ and consider

$$z = f(x, y)$$

Think of this as a surface in $\mathbb{R}^3$ (one height $z$ for each pt $(x, y)$ in $xy$ plane where $fx, fy$ make sense)

Domain: $D$, a set given where $f(x, y)$ is meaningful or the set of all $(x, y)$

Range: $R$, set of all $z$ values found from $z = f(x, y)$ with $(x, y)$ in $D$.

Ex: $f(x, y) = \frac{\sqrt{x+y+1}}{x-1}$ Domain?

$x \neq 1$, $x+y+1 \geq 0$

Ex Domain? Range? for $g(x, y) = \sqrt{9-x^2-y^2}$