Due Wednesday, May 7, 2014

Students in section B13 (three credit hours) need to solve any three of the following four problems. Students in section B14 (four credit hours) must solve all four problems.

For let first two problems, use the preceding graph and write down the sequence of edges that are added to the tree in the proper order.

1. Solve the minimum spanning tree problem (a) using Prim’s algorithm starting from the vertex $k$ and (b) using Kruskal’s algorithm.

2. Find a spanning tree of maximum weight (a) using Prim’s algorithm starting from the vertex $k$ and (b) using Kruskal’s algorithm.

3. For positive integers $k_1, \ldots, k_s$, let $E_1 = \{e_{1,1}, \ldots, e_{1,k_1}\}, \ldots, E_s = \{e_{s,1}, \ldots, e_{s,k_s}\}$ be disjoint finite sets and $E = \bigcup_{i=1}^{s} E_i$. Let $d_1, \ldots, d_s$ be positive integers. Let $\mathcal{I}$ be the family of subsets $M$ of $E$ such that $|M \cap E_i| \leq d_i$ for every $i$. Prove that $M = (E, \mathcal{I})$ is a matroid.

4. What are the bases and circuits of the matroid in the previous problem?