Due Friday, February 14, 2014

Students in section B13 (three credit hours) need to solve any four of the following five problems. Students in section B14 (four credit hours) must solve all five problems.

1. Solve the problem on the tableau using the simplex method and draw the corresponding picture in the plane $Ox_1x_2$. Mark the points corresponding to the b.f.s. of your solution.

\[
\begin{array}{cccccc}
-x & x_1 & x_2 & x_3 & x_4 & x_5 \\
0 & -3 & -4 & 0 & 0 & 0 \\
x_3 & 6 & 2 & 1 & 1 & 0 \\
x_4 & 2 & 1 & -2 & 0 & 1 \\
x_5 & 1 & -3 & 9 & 0 & 0 & 1 \\
\end{array}
\]

2. Introduce 3 artificial variables and solve with two-phase simplex algorithm the LP represented by the tableau below.

\[
\begin{array}{cccccc}
-x & x_1 & x_2 & x_3 & x_4 & x_5 \\
0 & 4 & 8 & 14 & 2 & 10 \\
x_4 & 14 & 2 & 2 & 4 & 2 \\
x_5 & 12 & 2 & 4 & 6 & 2 \\
x_6 & 8 & 2 & 2 & 2 & 4 \\
\end{array}
\]

3. Suppose that at a stage of the simplex algorithm, we have the following tableau $\tilde{T}$:

\[
\begin{array}{cccccccc}
-x & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\
8 & 0 & 8/3 & -11 & 0 & 4/3 & 0 \\
x_1 & 4 & 1 & 2/3 & 0 & 0 & 4/3 & 0 \\
x_4 & 2 & 0 & -7/3 & 3 & 1 & -2/3 & 0 \\
x_6 & 2 & 0 & -2/3 & -2 & 0 & 2/3 & 1 \\
\end{array}
\]

The inverse of the current basis is

\[
B^{-1} = [A_1, A_4, A_6]^{-1} = \frac{1}{3} \begin{pmatrix}
1 & 1 & -1 \\
1 & 1 & 2 \\
-2 & 2 & 1 \\
\end{pmatrix}
\]

and

\[
c_B^T = [c_1, c_4, c_6] = [-1, -3, 1].
\]

Find vectors $c$ and $b$ and the matrix $A$ that correspond to the original linear program.
4. Solve the LP in Example 2.7 (pages 51–52) of the book by Bland’s anticycling algorithm and by lexicographic simplex.

5. For both version of the simplex algorithm used in Problem 4, find the matrix by which we have to pre-multiply the original tableau in order to get the final tableau. In other words, if $T$ is the initial tableau and $\tilde{T}$ is the final tableau, then find the matrix $X$ such that $XT = \tilde{T}$. You should use the fact that $XT_i = \tilde{T}_i$ where $T_i$ is column $i$ of $T$ and $\tilde{T}_i$ is column $i$ of $\tilde{T}$. 