Due Friday, April 15, 2016

Students in section E13 (three credit hours) need to solve any four of the following five problems. Students in section E14 (four credit hours) must solve all five problems.

1. Beginning with the vector $\pi = (0, 0, 0, 0, 0)^T$ which is feasible for the dual, use the primal dual method as described in class and section 5.4 of the book to find a shortest path from $s$ to $t$ in the weighted graph shown below. For each iteration, you must write $\pi$, $\pi^r$ and $\theta$.

2. Apply the Ford-Fulkerson algorithm to the following network. On each iteration, write the current flow $f$ (starting with $f = 0$), the augmenting path $f^r$ and $\theta$ (you only need to write the non-zero elements of $f^r$).
3. A matching is a set of edges such that no vertex is contained in more than one edge. Using maximum flows, find a maximum matching in the bipartite graph below on the left. Prove that the matching is optimal. *Hint: Construct an flow network and then find a flow in this network that corresponds to a maximum matching in the graph. Show this flow is maximum by finding a minimum cut in the network. You should clearly write the flow network you are using, the max flow and the minimum cut in this network.*

4. Find the length of a shortest paths between every pair of vertices in the following directed graph using the Floyd–Warshall algorithm. Your answer should include the matrices \( D^j \) and \( E^j \) for every \( j \) from 0 to 4.

5. For the directed graph in the previous problem (problem 4), write the steps necessary to reconstruct the shortest path from node 4 to node 1 using the matrix \( E^4 \).