Math-484 Homework #9 (penalty method)

Due 11am Nov 19.

1: Consider the following program:

\[
(P) \begin{cases}
\text{Minimize} & f(x) = x^2 - 2x \\
\text{subject to} & 0 \leq x \leq 1.
\end{cases}
\]

(a) Sketch the graphs of the Absolute Value and Courant-Beltrami Penalty Terms for \((P)\).
(b) For each positive integer \(k\), compute the minimizer \(x_k\) of the corresponding unconstrained objective function \(F_k(x)\) with the Courant-Beltrami Penalty Term.
(c) For each positive integer \(k\), compute the minimizer \(x_k\) of the corresponding unconstrained objective function \(F_k(x)\) with the Absolute Value Penalty Term.

2:

a) Use the penalty function method with the Courant-Beltrami penalty term to solve the problem \((P)\).

\[
(P) \begin{cases}
\text{Minimize} & f(x_1, x_2) = x_1 + x_2 \\
\text{subject to} & x_1^2 - x_2 \leq 2
\end{cases}
\]

b) Show that the objective function \(F_k(x)\) corresponding to the Absolute value penalty term has no critical points off the parabola

\[x_1^2 - x_2 = 2\]

for \(k > 1\) and compute the minimizer of \(F_k(x)\).

3: Use the Penalty Function Method with Courant-Beltrami Penalty Term to minimize

\[f(x, y) = x^2 + y^2\]

subject to constraint \(x + y \geq 1\).

4: Let \(\varepsilon > 0\). Show that if a vector \(\lambda\) is a feasible for the dual \((D)\) of a convex program \((P)\), then \(\lambda\) is also feasible for \((D^\varepsilon)\).

5: We know that if a convex program is superconsistent, then \(MP = MD\). Show that the converse is not true. That is: find a convex program that is not superconsistent and yet \(MP = MD\).

Hint: You can use the fact the for all linear programs \(MP = MD\).

6: \((D14\ only)\) Let \(g(x)\) be a differentiable function on \(\mathbb{R}^1\) and suppose that \(g(x_0) = 0\) for some \(x_0 \in \mathbb{R}^1\).

(a) Show that \(g^+(x)\) is differentiable at \(x_0\) if and only if \(g'(x_0) = 0\).

(b) Show carefully that \((g^+(x))^2\) is differentiable at \(x_0\) and that its derivative at \(x_0\) is zero.