Math-484 Homework #7 (KKT conditions)

**Due 11am Nov 7.**
Write your name on your solutions and indicate if you are a D14 (4 credit hour) student.

1: Let \((P)\) be a convex program and let

\[ S := \{ x \in C : f(x) = MP \text{ and } g_i(x) \leq 0 \text{ for all } i \in [n] \}. \]

Prove that \(S\) is a convex set.
*Hint:* First show that if \(f\) is convex on \(C\), then for any \(m \in \mathbb{R}\), the set \(\{ x \in C : f(x) \leq m \}\) is convex.

2: Apply the Karush-Kuhn-Tucker Theorem to locate all solutions of the following convex programs:

\[
(P_a) \begin{cases}
\text{Minimize} & f(x_1, x_2) = e^{-(x_1+x_2)} \\
\text{subject to} & e^{x_1} + e^{x_2} \leq 20 \\
& x_1 \geq 0
\end{cases}
(P_b) \begin{cases}
\text{Minimize} & f(x_1, x_2) = x_1^2 + x_2^2 - 4x_1 - 4x_2 \\
\text{subject to} & x_1^2 - x_2 \leq 0 \\
& x_1 + x_2 \leq 2
\end{cases}
\]

3: Apply the Karush-Kuhn-Tucker Theorem to locate all solutions of the following convex program:

\[
(P) \begin{cases}
\text{Minimize} & -x_1 + x_2 \\
\text{subject to} & x_1^2 + x_1 - x_2 - 2 \leq 0 \\
& 11x_1 + 5x_2 - 6 \leq 0
\end{cases}
\]

4: Consider the following geometric program:

\[
(GP) \begin{cases}
\text{Minimize} & f(t_1, t_2) = t_1^{-1}t_2^{-1} \\
\text{subject to} & \frac{1}{2}t_1 + \frac{1}{2}t_2 \leq 1 \\
& \text{where } t_1 > 0, t_2 > 0
\end{cases}
\]

a) Convert \((GP)\) to an equivalent convex program and solve the resulting program using KKT.
b) Solve the given \((GP)\) by using methods of Chapter 5.3.

5: Solve the following geometric program:

\[
(GP) \begin{cases}
\text{Minimize} & x^{1/2} + y^{-2}z^{-1} \\
\text{subject to} & x^{-1}y^2 + x^{-1}z^2 \leq 1 \\
& \text{where } x > 0, y > 0, z > 0
\end{cases}
\]
Let $A$ be an $m \times n$ matrix and let $b \in \mathbb{R}^m$ be a fixed vector. Suppose that the convex program

\[
(P) \begin{cases} 
\text{Minimize} & \|x\|^2 \\
\text{subject to} & Ax \leq b 
\end{cases}
\]

is superconsistent and has a solution $x^*$. Use Karush-Kuhn-Tucker Theorem to show that there is a vector $y \in \mathbb{R}^m$ such that $x^* = A^T y$. 

6: (D14 only)