Math-484 Homework #6 (least squares, convex sets)

Due 11am Oct 22.
Write your name on your solutions and indicate if you are a D14 (4 credit hour) student.

1: Compute the best least square fit for polynomial

\[ p(t) = x_0 + x_1 t + x_2 t^2 \]

and data

<table>
<thead>
<tr>
<th>( t_i )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_i )</td>
<td>-5</td>
<td>-1</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>-1</td>
</tr>
</tbody>
</table>

2: Find best least squares solution to inconsistent linear system using QR factorization.

\[
\begin{pmatrix}
1 & 1 & 2 \\
1 & 4 & 6 \\
1 & 1 & 0 \\
1 & 4 & 4
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix} =
\begin{pmatrix}
4 \\
4 \\
2 \\
2
\end{pmatrix}
\]

3: (a) Find the point on the plane

\[ x + 2y + z = 6 \]

that is closest to the origin.

(b) Find the minimum norm solution of the underdetermined linear system

\[
2x_1 + x_2 + x_3 + 5x_4 = 8 \\
-x_1 - x_2 + 3x_3 + 2x_4 = 0
\]

4: Find vector \( x \in \mathbb{R}^3 \) that is closest to \((1, 1, 1)\) where \( \alpha, \beta \in \mathbb{R} \) and

\[ x = \alpha (1, 1, 2) + \beta (2, -1, 1) \]

5: Let \( C \) be a closest convex subset of \( \mathbb{R}^n \). If \( y \not\in C \), show that \( x^* \in C \) is the closest vector to \( y \) in \( C \) if and only if \( (x - y)^T (x^* - y) \geq ||x^* - y||^2 \) for all \( x \in C \).

6: Prove that if \( M \) is a subspace of \( \mathbb{R}^n \) such that \( M \neq \mathbb{R}^n \), then the interior \( M^0 \) of \( M \) is empty.

Hint: Use that the orthogonal complement \( M^\perp \) of \( M \) is also a subspace. Recall \( M^\perp = \{ x \in \mathbb{R}^n : x^T y = 0 \text{ for all } y \in M \} \). You should use the fact that \( M^\perp \cap M = \{0\} \).