Math-484 List of definitions and theorems

Definitions (Midterm 1):
- cosine of two vectors page 6
- distance of two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ page 6
- ball $B(\mathbf{x}, r)$ (what is $\mathbf{x}$ and $r$?) page 6
- interior $D^0$ of set $D \subseteq \mathbb{R}^n$ page 6, page 164
- open set $D \subseteq \mathbb{R}^n$ page 6
- closed set $D \subseteq \mathbb{R}^n$ page 7
- compact set $D \subseteq \mathbb{R}^n$ page 6
- (global,local)(strict) minimizer and maximizer of a function $f : \mathbb{R}^n \to \mathbb{R}$ page 8
- critical point of a function $f : \mathbb{R}^n \to \mathbb{R}$ page 8
- gradient $\nabla f(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ page 10
- Hessian $H f(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ page 10
- quadratic form associated with a symmetric matrix $A$ page 12
- (positive,negative)(semi)definite matrix page 13
- indefinite matrix page 13
- saddle point of a function $f : \mathbb{R}^n \to \mathbb{R}$ page 23
- $\Delta_k$, the $k^{th}$ principal minor of a matrix $A$ page 16
- coercive functions page 25
- eigenvalues and eigenvectors of a matrix $A$ page 29
- convex sets in $\mathbb{R}^n$ page 38
- closed and open half-spaces in $\mathbb{R}^n$ page 40
- convex combination of $k$ vectors from $\mathbb{R}^n$ page 41
- convex hull of $D \subseteq \mathbb{R}^n$ page 42
- (strictly) convex and concave function $f : C \to \mathbb{R}$, where $C \subseteq \mathbb{R}^n$ page 49
Theorems and statements (for Midterm 1):

(Try to not ignore assumptions - like that sometimes the function must be continuous etc.)

Proofs are only for D14 (4 credit hour) students

- State Cauchy-Swartz inequality (page 6)
- Minimizers and maximizers of continuous function $f : I \to \mathbb{R}$ where $I \subset \mathbb{R}$ is a closed interval (Theorem 1.1.4)
- local minimizers and the gradient (Theorem 1.2.3)
- Taylor’s formula for $\mathbb{R}^n$ (Theorem 1.2.4)
- $H_f$ and global minimizers and maximizers? (Theorem 1.2.5/Theorem 1.2.9)
- Principal minors of matrix $A$ and there relation to positive(negative) (semi)definite matrices $A$ (Theorem 1.3.3)
- Eigenvalues of a symmetric matrix and there relation to positive(negative) (semi)definite matrices (Theorem 1.5.1)
- $H_f$ and local minimizers and maximizers. (Theorem 1.3.6, with proof)
- coercive functions and minimization (Theorem 1.4.4, with proof)
- The convex hull of $D \subseteq \mathbb{R}^n$, $co(D)$, is the set of all convex combinations of vectors from $D$. ($D \subseteq \mathbb{R}^n$) (Theorem 2.1.4)
- convex function and continuity (Theorem 2.3.1)
- minimizers of convex functions (Theorem 2.3.4 with proof)
- maximizers of concave functions (Theorem 2.3.4)
- the relationship between convex function and the gradient (Theorem 2.3.5)
- critical points of convex function and minimization (Theorem 2.3.5 + Corollary 2.3.6)
- The relationship between the Hessian and convexity of a function (in $\mathbb{R}^n$) (Theorem 2.3.7)
- building convex function from other convex functions (Theorem 2.3.10)
- inequality involving convex functions and convex combinations with the condition for equality (finite version of Jensen’s Inequality) (Theorem 2.3.3)
- arithmetic-geometric mean inequality with the condition for equality (Theorem 2.4.1 with proof)