Quasirandom Hypergraphs

John Lenz   Dhruv Mubayi

University of Illinois at Chicago

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Quasirandom Graphs

Theorem

Early work by Thomason (1987) and Chung-Graham-Wilson (1989). Fix $0 < p < 1$, $\mathcal{G} = \{G_n\}_{n \to \infty}$ *with* $|V(G_n)| = n$ *and* $|E(G_n)| \geq p\binom{n}{2} + o(n^2)$. *The following are equivalent.*

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- *Count[*C_4*]: The number of labeled* C_4*s in* G *is* $p^4n^4 + o(n^4)$,
- *Eig*: $|\lambda_1| = pn + o(n)$ *and* $|\lambda_2| = o(n)$.

From k-SAT to Hypergraphs

Definition

A *k-uniform hypergraph* is a pair of sets $(V(H), E(H))$ where $E(H)$ is a collection of k-subsets of $V(H)$.

From a 3-SAT formula, build a 3-uniform hypergraph.

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_1 \vee \neg x_3 \vee x_4)$$

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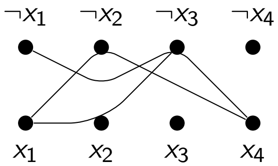

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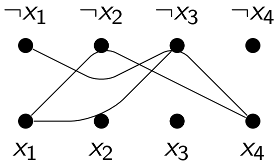

- Each truth assignment corresponds to a partition of the vertices. For example, $x_1 = T, x_2 = F, x_3 = T, x_4 = T$ corresponds to the vertex sets $T = \{x_1, \neg x_2, x_3, x_4\}$ and $F = \{\neg x_1, x_2, \neg x_3, \neg x_4\}$.

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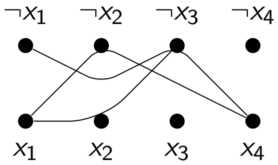

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- The assignment is a satisfying assignment if and only if F is a strong independent set.

Observation (Rödl)

For k-uniform hypergraphs with $k \geq 3$, $\texttt{Disc} \not\Rightarrow \texttt{Count}[\text{All}]$

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Proof.

Use Erdős and Hajnal's construction: let T be a random graph tournament and form a three-uniform hypergraph by making each cyclically oriented triangle a hyperedge. There is no $K_4^{(3)}$ but $\texttt{Disc}$ holds. □

Strong Hypergraph Quasirandomness

Theorem (Chung-Graham 1990, Kohayakawa-Rödl-Skokan 2002)

Fix $k \geq 2$, $0 < p < 1$, $\mathcal{H} = \{H_n\}_{n \to \infty}$ where H_n is k-uniform, $|V(H_n)| = n$, and $|E(H_n)| = p\binom{n}{k} + o(n^k)$. The following are equivalent.

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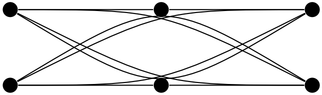
- $\mathtt{CliqueDisc}[k-1]$
- $\mathtt{Count}[All]$
- $\mathtt{Count}[Octahedron]$

The *octahedron* is $K(\underbrace{2, \ldots, 2}_{k})$.



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- *Disc*: $\forall U \subseteq V(H), |E(H[U])| = p\binom{|U|}{k} + o(n^k)$.
- *Expand[$1 + \cdots + 1$]*: $\forall S_1, \ldots, S_k \subseteq V(H)$,

$$e(S_1, \ldots, S_k) = p \prod |S_i| + o(n^k),$$

  where $e(S_1, \ldots, S_k)$ is the number of tuples $(s_1, \ldots, s_k) \in S_1 \times \cdots \times S_k$ where $\{s_1, \ldots, s_k\} \in E(H)$.

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- *Count[linear]. A hypergraph F is linear if $\forall E_1, E_2 \in E(F)$, $|E_1 \cap E_2| \leq 1$.*
- *Count[C_4].*

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Start with a single edge.

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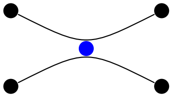
Reflect around the middle vertex.



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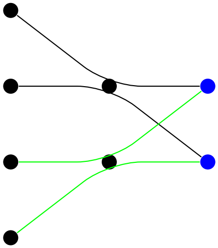
Reflect around the last two vertices.



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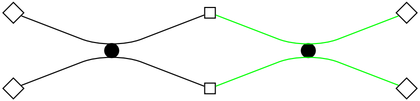
Straighten out the drawing and call it a step.

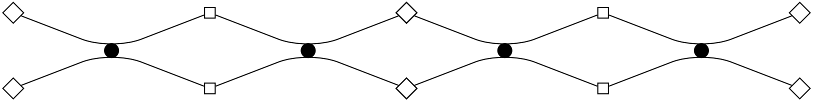


(The squares, circles, and diamonds keep track of the parts)

Discrepency and Counting - Definitions

For $k = 3$, C_4 is built as follows.

Connect steps into long paths.



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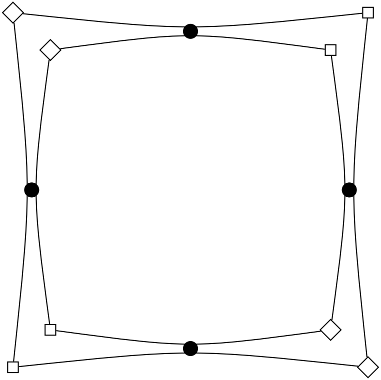
Identify the endpoints of the path.



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- Introduced Eig for hypergraphs.
- Generalized Kohayakawa-Nagle-Rödl-Schacht and Conlon-Hàn-Person-Schacht substantially.
- Determined the relationships between all studied hypergraph quasirandom properties, thereby completing a project begun by Chung in 1990.

Generalized Expansion

- For $k = 3$, $\texttt{Expand}[1+1+1]$: $\forall R, S, T \subseteq V(H)$,

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- $\texttt{Expand}[1+1+1] \Leftrightarrow \texttt{Disc}$
- $\texttt{Expand}[2+1] \Rightarrow \texttt{Expand}[1+1+1]$ but the converse is false!
- More generally for $k \geq 2$, let $\pi = k_1 + \cdots + k_t$ be a proper partition of k and define $\texttt{Expand}[\pi]$ as follows: $\forall S_i \subseteq \binom{V(H)}{k_i}$,

$$e(S_1, \ldots, S_t) = p|S_1| \cdots |S_t| + o(n^k).$$

Our Theorem

Theorem (Lenz-Mubayi 2012+)

Fix $k \geq 2$, $\pi = k_1 + \cdots + k_t$, and $0 < p < 1$. Let $\mathcal{H} = \{H_n\}_{n \to \infty}$ where H_n is k-uniform, $|V(H_n)| = n$, and $|E(H_n)| \geq p\binom{n}{k} + o(n^k)$. The following are equivalent.

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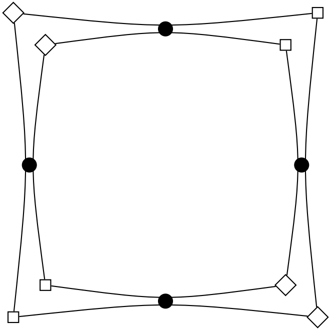
- *Count[$C_{\pi,4}$]*,
- *Count[$C_{\pi,4\ell}$]*.

Definition

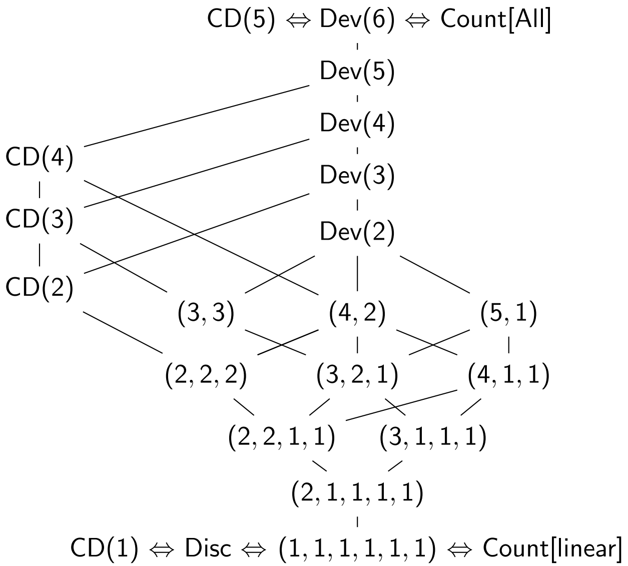
Let $k \geq 2$ and let $\pi = k_1 + \cdots + k_t$ be a proper partition of k. A k-uniform hypergraph F is π-*linear* if there exists an ordering $E_1, \ldots, E_m$ of the edges of F such that for every i, there exists a partition of the vertices of E_i into $A_{i,1}, \ldots, A_{i,t}$ such that for $1 \leq s \leq t$, $|A_{i,s}| = k_s$ and for every $j < i$, there exists an s such that $E_j \cap E_i \subseteq A_{i,s}$.

$C_{\pi,4}$

For the cycle $C_{\pi,4}$, start with C_4 defined earlier for t-uniform and expand. For $t = 3$, each diamond vertex is expanded into k_1 vertices, each square vertex into k_2 vertices, and each circle vertex into k_3 vertices.



Six Uniform Hypergraph Quasirandom Properties



Poset of Hypergraph Quasirandomness

Theorem (Lenz-Mubayi 2012+)

For every $k \geq 2$, $1 \leq \ell < k$, and every π, the poset of all hypergraph quasirandom properties involving $\texttt{Expand}[\pi]$, $\texttt{Deviation}[\ell]$, and $\texttt{CliqueDisc}[\ell]$ is completly determined.

Hypergraph Eigenvalues

What is $\lambda_{1,\pi}(H)$ and $\lambda_{2,\pi}(H)$? We base our definitions on definitions of Friedman and Widgerson (1995). Fix $k = 3$ and $\pi = 1 + 1 + 1$.

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- Let $W = \mathbb{R}^n$ be the vector space of dimension $n = |V(H)|$.
- The *adjacency map of H* is

$$\tau : W \times W \times W \to \mathbb{R}$$

$$\tau(e_x, e_y, e_z) = \begin{cases} 1 & \text{if } \{x, y, z\} \in E(H) \\ 0 & \text{otherwise} \end{cases}$$

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- $\lambda_{1,(1+1+1)}(H)$ is the *spectral norm* of τ,

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- $\lambda_{2,(1+1+1)}(H)$ is

$$\left\| \tau - \frac{k!E(H)}{n^k} J \right\|,$$

where J is the all-ones map.

Certifying Hypergraph Quasirandomness

Algorithm

On input k, ϵ, η, and H, let N be the number of labeled $C_{\pi,4\ell}$ in H, where $\pi = \lceil k/2 \rceil + \lfloor k/2 \rfloor$ and $\ell = \lceil k/(4\epsilon) \rceil$ if k is even and $\ell = \left\lceil (k+1)/(4\sqrt{k} - 2) \right\rceil$ if k is odd. Output `Quasirandom` if

$$N < (1+\eta)(k!)^{4\ell} |E(H)|^{4\ell} |V(H)|^{-2\ell k}.$$

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Theorem (Lenz-Mubayi 2012+)

$\forall \epsilon \exists \eta$ so that the following holds.

- *If the algorithm outputs `Quasirandom` then* *Expand*$[\lceil k/2 \rceil + \lfloor k/2 \rfloor]$ *holds.*
- *With high probability, the algorithm outputs `Quasirandom` on $G^{(k)}(n,p)$ if $p \geq n^{-k/2+\epsilon}$ for k even and $p \geq n^{k/2+\sqrt{k}}$ for k odd.*

Refuting random k-SAT

Algorithm

On input k, ϵ, η, and a k-SAT formula ϕ, convert ϕ to a hypergraph H and run the previous algorithm counting $C_{\pi,4\ell}$ in H. If the algorithm outputs `Quasirandom`, output "ϕ is `Unsatisfiable`", otherwise output `Unknown`.

Refuting random k-SAT

Algorithm

On input k, ϵ, η, and a k-SAT formula ϕ, convert ϕ to a hypergraph H and run the previous algorithm counting $C_{\pi,4\ell}$ in H. If the algorithm outputs `Quasirandom`, output "ϕ is `Unsatisfiable`", otherwise output `Unknown`.

Theorem (Lenz-Mubayi 2012+)

$\forall \epsilon \exists \eta$ *so that the following holds*

- *If the algorithm outputs* `Unsatisfiable`*, then the formula is unsatifiable.*
- *With high probability, the algorithm outputs* `Unsatisfiable` *on* $Form_k(n, p)$ *if* $p \geq n^{-k/2+\epsilon}$ *for k even and* $p \geq n^{k/2+\sqrt{k}}$ *for k odd*

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- Can some graph Ramsey results that rely on graph eigenvalues be translated to hypergraphs now that we know $\texttt{Eig}[\pi]$?