Math 484: Nonlinear Programming — Mikhail Lavrov

Homework #8

Spring 2019 — *Due Friday, April 5*

1. Solve the following problem by using KKT duality:

$$\begin{aligned} \underset{(x,y)\in\mathbb{R}^2}{\text{minimize}} \quad & x^2 + 2y \\ \text{subject to} \quad & x + y \geq 3. \end{aligned}$$

2. Solve the following geometric program:

$$\begin{aligned} \underset{(x,y,z)\in\mathbb{R}^3}{\text{minimize}} \quad & \frac{1}{x} + \frac{2}{y} + \frac{4}{z} \\ \text{subject to} \quad & xyz \leq 1, \\ & x > 0, y > 0, z > 0. \end{aligned}$$

3. Among the constraints below, four can be expressed using posynomial constraints (constraints that can appear in a geometric program), and one cannot.

   For the ones that can, show how, and say which one is impossible.

   (a) $xy \geq 1$

   (b) $x^2 + y^2 \leq z^2$

   (c) $x^2 - y^2 \geq z^2$

   (d) $x - y \leq 1$

   (e) $x^2 y^{-1} = 1$

4. Solve the optimization problem

$$\begin{aligned} \underset{x,y\in\mathbb{R}}{\text{minimize}} \quad & x + y^2 \\ \text{subject to} \quad & x^2 - y^2 + 1 \leq 0 \end{aligned}$$

   by turning the constraint into a quadratic penalty term, minimizing

$$x + y^2 + M(\max\{0, x^2 - y^2 + 1\})^2,$$

   and taking a limit as $M \to \infty$.

   (Hint: one of the critical points of the penalized function has $y = 0$, but that can't possibly converge to an optimal solution—why?)