Math 484: Nonlinear Programming — Mikhail Lavrov

Homework #4

Spring 2019 — *Due Friday, February 22*

1. Consider the unconstrained geometric program

$$\begin{aligned} \underset{t_1,t_2\in\mathbb{R}}{\text{minimize}} \quad & \frac{t_1}{t_2} + t_2^4 + \frac{4}{t_1 t_2} \\ \text{subject to} \quad & t_1, t_2 > 0. \end{aligned}$$

   (a) Write down the dual geometric program.

   (b) Find the dual optimal solution.

   (c) Solve for the primal optimal solution.

2. Consider the unconstrained geometric program[1]

$$\begin{aligned} \underset{t\in\mathbb{R}}{\text{minimize}} \quad & t^2 + \frac{1}{t} + 3t \\ \text{subject to} \quad & t > 0. \end{aligned}$$

   (a) Write down the dual geometric program.

   (b) Reduce the dual program to a 1-parameter optimization problem of the form "maximize $f(s)$ for s in some open interval", as in example (2.5.5)(c) in the textbook.

   Don't solve that optimization problem, though, because it looks painful.[2]

   (c) Assuming that the optimal dual solution is $\boldsymbol{\delta} = (\frac{1}{15}, \frac{8}{15}, \frac{2}{5})$, find the optimal value of t in the primal program.

3. Consider the geometric program

$$\begin{aligned} \underset{t_1,t_2\in\mathbb{R}}{\text{minimize}} \quad & t_1 t_2 + \frac{1}{t_1 t_2^2} \\ \text{subject to} \quad & t_1, t_2 > 0. \end{aligned}$$

   (a) Find the dual program, and show that it's infeasible.

   (b) Find a way to make the objective value of the primal program arbitrarily close to 0.

4. Find the line of best fit through the points

$$\{(-1,2),(0,1),(1,3),(2,2),(3,1)\}.$$

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[1] Yes, I realize that there are easier ways to solve this problem than by treating it as a geometric program.

[2] If you wanted to solve it, you'd start with your $v(\boldsymbol{\delta}) = f(s)$, take the derivative of $\log f(s)$, collect everything inside the log and set it equal to 1, and then solve the cubic equation you get to determine s: this will be a critical point of $\log f(s)$, and $\log f(s)$ is concave, so the critical point will maximize $\log f(s)$, so it will maximize $f(s)$. It's not impossible. Just annoying.