Math 484: Nonlinear Programming — Mikhail Lavrov

Homework #3

Spring 2019 — *Due Friday, February 15*

1. Show that the "diamond" set $D = \{(x, y) : |x| + |y| \leq 1\}$ is convex. (One way to do this, though not the only way, is to verify that D satisfies Definition 2.1.1 in the textbook.)

2. Give an example of each of the following, with justification:

   (a) Functions $f, g : \mathbb{R} \to \mathbb{R}$ such that f and g are convex, but the composition $h(x) = g(f(x))$ is not convex.

   (b) Functions $f, g : \mathbb{R} \to \mathbb{R}$ such that f is convex and g is increasing, but the composition $h(x) = g(f(x))$ is not convex.

   (c) Functions $f, g : \mathbb{R} \to \mathbb{R}$ such that f and g are convex, but the product $h(x) = f(x)g(x)$ is not convex.

3. Show that the functions are convex on the indicated sets.

   (a) $f(x) = e^{e^{e^x}}$ on $\mathbb{R}$.

   (b) $f(x, y, z) = (x + 2y)^4 + (y - z)^4$ on $\mathbb{R}^3$.

   (c) $f(x) = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$ on $\mathbb{R}$.

4. Use the AM-GM inequality to solve this optimization problem:

$$\begin{aligned} \underset{x,y,z \in \mathbb{R}}{\text{minimize}} \quad & xy^2 + yz^2 + zx^2 \\ \text{subject to} \quad & xyz = 1, \\ & x, y, z > 0. \end{aligned}$$

5. *(Only 4-credit students need to do this problem.)*

   Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function. Show that the sublevel set

$$L_c^-(f) = \{\mathbf{x} \in \mathbb{R}^n : f(\mathbf{x}) \leq c\}$$

   is a convex set.