1. Solve the following convex program:

\[
\begin{align*}
\text{minimize} & \quad x + y \\
\text{subject to} & \quad x^2 - y \leq 0, \\
& \quad y - 2x \leq -1.
\end{align*}
\]

2. Solve the following geometric program:

\[
\begin{align*}
\text{minimize} & \quad 2x^2 + 3y^{-2}z^{-1} \\
\text{subject to} & \quad x^{-1}y^2 + 2x^{-1}z^2 \leq 1, \\
& \quad x > 0, y > 0, z > 0.
\end{align*}
\]

3. Transform each of the constraints below into one or more posynomial constraints (constraints that can appear in a geometric program), whenever it is possible.

(a) \(x^2 - y^2 \leq 2\)

(b) \(x^2y^{-2} \geq 2\)

(c) \(x^{1/2} + y^{1/2} \geq z^{1/2}\)

(d) \(x^{1/2} + y^{1/2} \leq z^{1/2}\)

(e) \(x^2y^{-1} = 3\)

4. Use the penalty function method with the Courant–Beltrami penalty term to minimize

\[
f(x, y) = x^2 + y^2
\]

subject to the constraint \(x + y \geq 1\).

**General instructions for writing up homework:**

- When writing up solutions, if you use a result from your textbook, say the result you’re using (by name, or theorem number, or whatever) and why it applies. E.g., “So \(f''(5) = 1\). By the second derivative test, since \(f''(5) > 0\), the critical point \(x = 5\) is a strict local minimizer.”

- You don’t need to show your work for routine computations, but if you get those wrong without showing your work, you’ll miss the opportunity for partial credit.

- Write proofs in complete sentences.