1. On the first day of class, we discussed optimization problems in the standard form

\[
\text{minimize } f(x_1, x_2, \ldots, x_n) \\
\text{subject to } g_i(x_1, x_2, \ldots, x_n) \leq 0, \\
\ldots, \\
g_m(x_1, x_2, \ldots, x_n) \leq 0.
\]

More general problems can be put in this form. Explain how you’d put the following kinds of problems into the form above:

(a) A problem that involved maximizing a function \( f(x_1, x_2, \ldots, x_n) \).
(b) A problem with a constraint \( g_i(x_1, x_2, \ldots, x_n) \leq c_i \) for an arbitrary constant \( c_i \).
(c) A problem with a constraint \( g_i(x_1, x_2, \ldots, x_n) = 0 \).

2. Find the local and global minimizers of the following functions:

(a) \( f(x) = x^2 + 2x \).
(b) \( f(x) = x^2 e^{-x^2} \).
(c) \( f(x) = x + \sin x \).

3. Compute the eigenvalues and eigenvectors of the matrix \[
\begin{bmatrix}
5 & -2 \\
-1 & 4
\end{bmatrix}.
\]

4. (a) Find the Hessian matrix of the function \( f(x, y) = e^{-x^2-y^2} \).
(b) Use it to classify the critical point \((0,0)\) as a (strict) local minimizer of \( f(x, y) \), (strict) local maximizer of \( f(x, y) \), or none of these.

5. (Only 4-credit students need to do this problem.)

The law of cosines states that for a triangle \( \triangle ABC \),

\[
AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos \angle BAC = BC^2.
\]

Suppose that \( A, B, C \) are points in \( \mathbb{R}^n \) with coordinates \( A = 0, B = x, C = y \) for some vectors \( x, y \in \mathbb{R}^n \).

State the law of cosines in terms of \( x \) and \( y \), then prove it.

**General instructions for writing up homework:**

- If you’re taking the class for 4 credits (as opposed to the default of 3), write this on your assignment so that it can be graded appropriately.
- When writing up solutions, if you use a result from your textbook, say the result you’re using (by name, or theorem number, or whatever) and why it applies. E.g., “So \( f''(5) = 1 \). By the second derivative test, since \( f''(5) > 0 \), the critical point \( x = 5 \) is a strict local minimizer.”
- You don’t need to show your work for routine computations, but if you get those wrong without showing your work, you’ll miss the opportunity for partial credit.
- Write proofs in complete sentences.