Math 484: Topics Covered in Exam 1

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The first exam will, broadly speaking, cover all the material covered in class through Friday, September 14$^{\text{th}}$. In this document, I go through each relevant section of the textbook and point out the things you should make sure to know from it.

To study for the exam, I suggest the following resources:

- Review the first two homework assignments. Moodle (`https://learn.illinois.edu/`) will have solution sets for each assignment each Monday. It's worth reading over them even if you solved the problems correctly to see if you missed shortcuts or alternative approaches.
- When reviewing material from a specific section, look at the examples in the textbook, which are labeled "**(*x.y.z*) Example.**", and try to work through or understand them on your own before checking against the book's solution.
- The exercises at the end of each chapter are pretty good too. Some of them develop extra theory that wasn't covered in that chapter, which occasionally means they get deeper into the material than we do in this class. If you have questions about how to solve any of the exercises, I'm happy to answer them.

  For Chapter 1 in particular, I recommend any and all of the exercises that involve finding local and global minimizers and maximizers of functions.

1 Unconstrained Optimization via Calculus

1.1 Functions of One Variable

You should certainly understand the material in this section, and be able to do things like take derivatives, but this section is mostly a review of single-variable calculus. Any relevant definitions and results here are special cases of results in later sections.

1.2 Functions of Several Variables

You should know the definitions from this section, which there are a lot of:

- Vectors in $\mathbb{R}^n$, dot product, norm, and their properties.

- The open ball $B(\mathbf{x}, r)$ and the line segment $[\mathbf{x}, \mathbf{y}]$. (We didn't talk about line segments until later, but their definition is here on p. 11.)
- Interior, exterior (not defined in the textbook, but the exterior of S is the interior of the complement $\mathbb{R}^n \setminus S$) and boundary (not defined in the textbook: this is all the points neither in the interior nor the exterior.)
- Open sets, closed sets, and bounded sets. (Our definition of closed sets given in class differs from the textbook's slightly, but it is equivalent.)
- Minimizers and maximizers: local and global ones, strict and non-strict ones, in all combinations.
- Critical points.
- Quadratic forms of matrices.
- Positive semidefinite, positive definite, negative semidefinite, negative definite, and indefinite matrices and quadratic forms.

You should be able to use Theorem 1.2.3 (critical points on $\mathbb{R}^n$), Theorem 1.2.9 (the Hessian matrix test for global optimizers) and Theorem 1.3.6 (the Hessian matrix test for local optimizers) to find critical points and classify them as local minimizers, local maximizers, or neither.

1.3 Positive and Negative Definite Matrices and Optimization

You should know and be able to use Theorem 1.3.3 (Sylvester's criterion) to check if a matrix is positive definite, negative definite, or neither.

You should also understand the general argument that when a quadratic form can be written as a sum of squares, it is positive semidefinite. In particular, when looking at a diagonal matrix, it should be immediately obvious to you if it is positive (semi)definite, negative (semi)definite, or indefinite.

1.4 Coercive Functions and Global Minimizers

You should know Theorem 1.4.1.

Regarding coercive functions: you should know their definition, and be able to show that a function is coexrcive in the most straightforward cases. You should look through the examples (a)–(f) on page 27, but a couple of these are more complicated than I expect you to handle on your own for now.

Knowing Theorem 1.4.4 is a potential resource for checking for global minimizers, and may make your life easier.

1.5 Eigenvalues and Positive Definite Matrices

You should know how to compute eigenvalues of a matrix, at least in theory. (In practice, computing eigenvalues of an $n \times n$ matrix exactly requires solving a degree n polynomial; I will not expect you to solve these except when they obviously factor, when the quadratic formula applies, or a combination of the two.)

You should know and be able to use Theorem 1.5.1 (the eigenvalue test) to check if a matrix is positive (semi)definite, negative (semi)definite, or neither.

I will leave it up to you as much as possible whether you should use the material in this section or in Section 1.3 to check the definiteness of matrices, but you should certainly understand how both of them work, and in practice one may sometimes be easier than the other.

2 Convex Sets and Convex Functions

2.1 Convex Sets

You should know the definition of a convex set and of the convex combination of points in $\mathbb{R}^n$, and you should know Theorem 2.1.3.

You should be able to prove that a set is convex, either from the definition or from the intersection property (which is example 2.1.2.f in the textbook).

2.2 Some Illustrations of Convex Sets in Economics

We skip Section 2.2. You don't need to know it.

2.3 Convex Functions

You should know the definition of a convex function (Definition 2.3.2) and the Hessian matrix test (Theorem 2.3.7).

You should know Theorem 2.3.3 (Jensen's inequality).

This section also covers other properties of convex functions and other ways to prove that a function is convex. We will talk about these on Monday, but you will not need to know these for the exam.