Math 484: Nonlinear Programming — Mikhail Lavrov

Homework #9

Fall 2018 — *Due Friday, November 30*

1. Use the penalty function method with the Courant–Beltrami penalty term to solve

$$\begin{aligned} \underset{x,y\in\mathbb{R}}{\text{minimize}} \quad & x^2 + xy + y^2 \\ \text{subject to} \quad & x - y \leq 2. \end{aligned}$$

2. Show that the following functions are coercive:

   (a) $f(x, y) = x^2 + y^2 + e^{x-y}$.

   (b) $f(x, y, z) = x^2 + y^4 + z^6 - 10xyz$.

   (c) $f(x, y) = x^2 + 5y^2 - 4xy$.

3. Put the constrained geometric program below in standard form, then solve it by using the dual geometric program.

$$\begin{aligned} \underset{x,y,z>0}{\text{minimize}} \quad & x^2 + y^2 + z \\ \text{subject to} \quad & xyz = 18. \end{aligned}$$

4. Starting at $x_0 = 1$, apply Newton's method for solving the equation $x^3 - x + 1 = 0$ to compute the first three iterations x_1, x_2, and x_3.

5. *(Only 4-credit students need to do this problem.)*

   Suppose we are using Newton's method to solve the equation $x^2 - 1 = 0$, starting from the initial guess $x_0 = 3$.

   Find a closed-form expression for the k^{th} iteration x_k of Newton's method.

   Think about the decay rate of the error $|x_k - 1|$ in terms of k.