Math 484: Nonlinear Programming		Mikhail Lavrov
	Homework #8	
Fall 2018		*Due Friday, November 9*

1. In this problem, you will see an example in which we can find an optimal solution using KKT duality, even though the program doesn't satisfy pretty much any of the conditions under which the KKT conditions are guaranteed to hold.

   Consider the problem

   $$(P) \qquad \begin{cases} \underset{x,y\in\mathbb{R}}{\text{minimize}} & x^2 - y^2 \\ \text{subject to} & (x-2y-2)^2 \le 0. \end{cases}$$

   (a) Find an explicit formula, in terms of λ, for the dual objective function

   $$h(\lambda) = \inf\{(x^2-y^2) + \lambda(x-2y-2)^2 : x, y \in \mathbb{R}\}.$$

   Find the values of (x, y), in terms of λ, for which the inf is achieved. Finally, determine the values of λ for which $h(\lambda) > -\infty$.

   (Hint: for a quadratic function, a local minimizer—if one exists—is always global.)

   (b) The dual problem has no optimal solution, but we can get arbitrarily close to the optimal value. By looking at what happens to the values (x, y) as we do so, find the optimal solution to (P).

2. Solve the constrained geometric program

   $$\begin{aligned} \underset{x,y,z>0}{\text{minimize}} \quad & x^2 + yz \\ \text{subject to} \quad & \frac{1}{y^2} + \frac{4}{xz^2} \le 1. \end{aligned}$$

3. Solve the optimization problem

   $$\begin{aligned} \underset{x,y\in\mathbb{R}}{\text{minimize}} \quad & x + y \\ \text{subject to} \quad & y \ge x^2 \end{aligned}$$

   by turning the constraint into a quadratic penalty term, minimizing

   $$x + y + M(\max\{0, x^2 - y\})^2,$$

   and taking a limit as $M \to \infty$.

4. *(Only 4-credit students need to do this problem.)*

   Consider the linear program

   $$(P) \qquad \begin{cases} \underset{\mathbf{x}\in S}{\text{minimize}} & \mathbf{c}\cdot\mathbf{x} \\ \text{subject to} & A\mathbf{x} \ge \mathbf{b}. \end{cases}$$

   where the domain of S is not all of $\mathbb{R}^n$ (as in the case we saw in class) but just $\{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} \ge \mathbf{0}\}$. That is, all components of $\mathbf{x}$ are required to be nonnegative.

   Find the KKT dual of P in this case.