Math 484: Nonlinear Programming | Mikhail Lavrov

Homework #6

Fall 2018 | *Due Wednesday, October 24*

1. Minimize $f(x, y, z) = 3x^2 - 6xy + 4y^2 + 6z^2$ subject to $x - y - z = 3$ again (you have already done it once on the previous homework assignment), but use Lagrange multipliers this time.

2. Let C be the convex set $\{(x, y) \in \mathbb{R}^2 : x^2 + y \leq 2 \text{ and } x + y^2 \leq 2\}$.

   Using the fact that $(1, 1)$ is the point of C closest to $(3, 4)$, find a linear inequality which is true for every point of C, false for $(3, 4)$, and holds with equality at $(1, 1)$.

3. For each of the following sets, determine their interior, boundary, and closure. You do not have to justify your answer.

   (a) $S_1 = \{1\} \cup [2, 3] \cup (4, 5) \subseteq \mathbb{R}$.

   (b) $S_2 = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + 3z = 4\}$.

   (c) $S_3 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ and } y \neq 0\}$.

4. Describe the subdifferential of the function $f(x) = x^2 + |x - 1| + |x - 2|$ at all points $x \in \mathbb{R}$.

5. *(Only 4-credit students need to do this problem.)*

   Let $C \subseteq \mathbb{R}^n$ be a convex set, and let $\mathbf{y}^{(1)}, \mathbf{y}^{(2)} \notin C$ be two points.

   Suppose that $\mathbf{0} \in C$, and moreover that $\mathbf{0}$ is the closest point of C to $\mathbf{y}^{(1)}$ *and* is the closest point of C to $\mathbf{y}^{(2)}$.

   Prove that $\mathbf{0}$ is also the closest point of C to $\frac{1}{2}\mathbf{y}^{(1)} + \frac{1}{2}\mathbf{y}^{(2)}$.