1. Use the Gram–Schmidt process to find orthonormal vectors $u^{(1)}, u^{(2)}, u^{(3)} \in \mathbb{R}^4$ that span the same subspace of $\mathbb{R}^4$ as

$$a^{(1)} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad a^{(2)} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 3 \end{bmatrix}, \quad a^{(3)} = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 6 \end{bmatrix}.$$

2. Let $u^{(1)}, u^{(2)}, \ldots, u^{(n)}$ be an orthonormal basis of $\mathbb{R}^n$, and let $v^{(1)}, v^{(2)}, \ldots, v^{(n)}$ be any vectors in $\mathbb{R}^m$. Show that the $m \times n$ matrix

$$M = v^{(1)}(u^{(1)})^T + v^{(2)}(u^{(2)})^T + \cdots + v^{(n)}(u^{(n)})^T$$

is the unique matrix with the property that $Mu^{(i)} = v^{(i)}$ for all $i$.

3. Find the minimum norm solution of the underdetermined linear system

$$x_1 + x_3 + x_4 = 1,$$

$$2x_1 + x_2 + x_3 + x_4 = 0.$$

4. Minimize $f(x, y, z) = 3x^2 - 6xy + 4y^2 + 6z^2$ subject to $x - y - z = 3$.

5. (Only 4-credit students need to do this problem.)

We say that an $n \times n$ matrix $P$ is a projection matrix if it satisfies $P^TP = P$.

(a) Given an arbitrary $m \times n$ matrix $A$, show that $AA^\dagger = A(A^TA)^{-1}A^T$ is a projection matrix by this definition.

(b) Given an arbitrary projection matrix $P$, show that for any vector $y \in \mathbb{R}^n$, $Py - y$ is orthogonal to any element of the subspace $\{Px : x \in \mathbb{R}^n\}$. 