Math 484: Nonlinear Programming | Mikhail Lavrov

Homework #4

Fall 2018 | *Due Friday, October 5*

1. Consider the unconstrained geometric program

$$\begin{aligned} \underset{t_1,t_2\in\mathbb{R}}{\text{minimize}} \quad & \frac{t_1}{t_2^3} + 2t_1t_2 + 4\frac{t_2}{t_1^3} \\ \text{subject to} \quad & t_1, t_2 > 0. \end{aligned}$$

   (a) Write down the dual geometric program.

   (b) Find the dual optimal solution.

   (c) Solve for the primal optimal solution.

2. Consider the geometric program

$$\begin{aligned} \underset{t_1,t_2,t_3\in\mathbb{R}}{\text{minimize}} \quad & \frac{t_1^2}{t_2t_3} + \frac{t_3^2}{t_1t_2} \\ \text{subject to} \quad & t_1, t_2, t_3 > 0. \end{aligned}$$

   (a) Find the dual program, and show that it's infeasible.

   (b) Show that the primal geometric program does not have an optimal solution $\mathbf{t}^*$. (What happens instead?)

3. Find the line of best fit through the points

$$\{(0,5),(1,1),(2,-1),(3,5)\}.$$

4. Find the lowest degree polynomial equation $y = P(x)$ that passes through the points

$$\{(0,5),(1,1),(2,-1),(3,5)\}.$$

5. *(Only 4-credit students need to do this problem.)*

   Recall that the null space of an $m \times n$ matrix A is the vector space $\{\mathbf{x} \in \mathbb{R}^n : A\mathbf{x} = \mathbf{0}\}$.

   Show that for any matrix A, the null space of $A^\mathsf{T}A$ is equal to the null space of A.