Math 484: Nonlinear Programming — Mikhail Lavrov

Homework #3

Fall 2018 — *Due Wednesday, September 26*

1. Show that the functions are convex on the indicated set.
   (a) $f(x) = |x^3|$ on $\mathbb{R}$.
   (b) $f(x_1, x_2, x_3) = (x_1 - x_2)^4 + (x_2 - x_3)^4$ on $\mathbb{R}^3$.
   (c) $f(x, y) = x^x y^y$ on $\{(x, y) \in \mathbb{R}^2 : x > 0, y > 0\}$.

2. Use the AM-GM inequality to solve this optimization problem:

$$\begin{aligned} \underset{x,y,z\in\mathbb{R}}{\text{minimize}} \quad & x^2 + y^2 + z \\ \text{subject to} \quad & xyz = 1, \\ & x, y, z > 0. \end{aligned}$$

   (Note that z is not squared!)

3. Use Jensen's inequality to derive the following inequality: if $x_1, x_2, \ldots, x_n$ are positive real numbers, then

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}.$$

   (The right-hand side of the inequality is known as the harmonic mean of $x_1, x_2, \ldots, x_n$: it is the reciprocal of the average of the reciprocals of $x_1, x_2, \ldots, x_n$.)

4. Write down the dual of the geometric program

$$\begin{aligned} \underset{x,y,z\in\mathbb{R}}{\text{minimize}} \quad & xy^2 + xyz + \frac{4yz^2}{x} \\ \text{subject to} \quad & x, y, z > 0. \end{aligned}$$

5. *(Only 4-credit students need to do this problem.)*

   Let $f : \mathbb{R}^n \to \mathbb{R}$ be a strictly convex function. Suppose that $\mathbf{x}$ and $\mathbf{y}$ are distinct points in $\mathbb{R}^n$ such that $f(\mathbf{x}) = f(\mathbf{y}) = 0$. Show that there is a $\mathbf{z} \in \mathbb{R}^n$ such that $f(\mathbf{z}) < 0$.