1. Show that the functions are convex on the indicated set.
   (a) $f(x) = |x^3|$ on $\mathbb{R}$.
   (b) $f(x_1, x_2, x_3) = (x_1 - x_2)^4 + (x_2 - x_3)^4$ on $\mathbb{R}^3$.
   (c) $f(x, y) = x^x y^y$ on $\{(x, y) \in \mathbb{R}^2 : x > 0, y > 0\}$.

2. Use the AM-GM inequality to solve this optimization problem:
   \[
   \begin{align*}
   \text{minimize} & \quad x^2 + y^2 + z \\
   \text{subject to} & \quad xyz = 1, \\
   & \quad x, y, z > 0.
   \end{align*}
   \]
   (Note that $z$ is not squared!)

3. Use Jensen’s inequality to derive the following inequality: if $x_1, x_2, \ldots, x_n$ are positive real numbers, then
   \[
   \frac{x_1 + x_2 + \cdots + x_n}{n} \geq \frac{1}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}.
   \]
   (The right-hand side of the inequality is known as the harmonic mean of $x_1, x_2, \ldots, x_n$: it is the reciprocal of the average of the reciprocals of $x_1, x_2, \ldots, x_n$.)

4. Write down the dual of the geometric program
   \[
   \begin{align*}
   \text{minimize} & \quad xy^2 + xyz + \frac{4yz^2}{x} \\
   \text{subject to} & \quad x, y, z > 0.
   \end{align*}
   \]

5. (Only 4-credit students need to do this problem.)
   Let $f : \mathbb{R}^n \to \mathbb{R}$ be a strictly convex function. Suppose that $x$ and $y$ are distinct points in $\mathbb{R}^n$ such that $f(x) = f(y) = 0$. Show that there is a $z \in \mathbb{R}^n$ such that $f(z) < 0$. 