Math 484: Nonlinear Programming	Mikhail Lavrov
Homework #2	
Fall 2018	*Due Friday, September 14*

1. Using any method you like, classify the matrices below as positive (semi)definite, negative (semi)definite, or indefinite.

$$\text{(a)}\ \begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{bmatrix} \quad \text{(b)}\ \begin{bmatrix} 2 & 1 & 4 \\ 1 & 3 & 1 \\ 4 & 1 & -1 \end{bmatrix} \quad \text{(c)}\ \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 3 & -6 & -7 \end{bmatrix}$$

2. Let B be the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

   (a) Find a 3×3 *symmetric* matrix A such that $\mathbf{x}^\mathsf{T} A \mathbf{x} = \mathbf{x}^\mathsf{T} B \mathbf{x}$.

   (b) Find the Hessian matrix of $\mathbf{x}^\mathsf{T} B \mathbf{x}$. (Because this is a quadratic function, the Hessian matrix, usually a function of $\mathbf{x}$, will be constant here.)

3. Let $f(x, y) = e^x + e^y + e^{1-x-y}$.

   (a) Show that the sublevel set

$$\{(x, y) \in \mathbb{R}^2 : f(x, y) \le L\}$$

   is contained in the triangle

$$\{(x, y) \in \mathbb{R}^2 : x \le \log L \text{ and } y \le \log L \text{ and } x + y \ge 1 - \log L\}.$$

   Conclude that $f(x, y)$ is coercive.

   (b) Find the global minimizer of f.

4. Show that the "diamond" set $D = \{(x, y) : |x| + |y| \le 1\}$ is convex. (One way to do this, though not the only way, is to verify that D satisfies Definition 2.1.1 in the textbook.)

5. *(Only 4-credit students need to do this problem.)*

   Let S be the set of all points $(x_{11}, x_{12}, x_{13}, x_{22}, x_{23}, x_{33}) \in \mathbb{R}^6$ such that the matrix

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{12} & x_{22} & x_{23} \\ x_{13} & x_{23} & x_{33} \end{bmatrix}$$

   is positive semidefinite. Is the set S convex? Justify your answer.