Math 484: Nonlinear Programming	Mikhail Lavrov
Homework #10	
Fall 2018	*Due Monday, December 10*

1. For the function $f(x, y) = x^2 + 2xy + 2y^2 + 4x - 2y$,
   (a) Perform one iteration of Newton's method for minimization, starting at $(0, 0)$.
   (b) Find the global minimizer of f.
2. Compute the first two iterations $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$ of the steepest descent method for minimizing the function

$$f(x_1, x_2) = 2x_1^4 + x_2^2 - 4x_1x_2 + 5x_2$$

   with initial point $\mathbf{x}^{(0)} = (0, 0)$.
3. Suppose that we want to use a descent method to minimize $f(x, y) = x^3 + y^3$ starting from the point $\mathbf{x}^{(0)} = (1, 2)$ in the direction $\mathbf{p}^{(0)} = (1, -1)$.

   Find the range of the values t_0 such that going from $\mathbf{x}^{(0)}$ to

$$\mathbf{x}^{(1)} = \mathbf{x}^{(0)} + t_0\mathbf{p}^{(0)}$$

   will satisfy the criteria of Wolfe's theorem, with constants $\alpha = \frac{1}{4}$ and $\beta = \frac{1}{2}$.
4. Determine the range of x for which

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & -2 \end{bmatrix} + x \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   is positive definite.
5. *(Only 4-credit students need to do this problem.)*
6. Let A be a symmetric $n \times n$ matrix. Show that if A is positive definite, then A^{-1} exists and is also positive definite.

General instructions for writing up homework:

- *If you're taking the class for 4 credits (as opposed to the default of 3), write this on your assignment so that it can be graded appropriately.*
- *When writing up solutions, if you use a result from your textbook, say the result you're using (by name, or theorem number, or whatever) and why it applies. E.g., "So $f''(5) = 1$. By the second derivative test, since $f''(5) > 0$, the critical point $x = 5$ is a strict local minimizer."*
- *You don't need to show your work for routine computations, but if you get those wrong without showing your work, you'll miss the opportunity for partial credit.*
- *Write proofs in complete sentences.*