Math 484: Nonlinear Programming — Mikhail Lavrov

Homework #1

Fall 2018 — *Due Friday, September 7*

1. On the first day of class, we discussed optimization problems in the standard form

$$\begin{aligned} \underset{x_1,x_2,\dots,x_n\in\mathbb{R}}{\text{minimize}} \quad & f(x_1,x_2,\dots,x_n) \\ \text{subject to} \quad & g_1(x_1,x_2,\dots,x_n) \le 0, \\ & \dots, \\ & g_m(x_1,x_2,\dots,x_n) \le 0. \end{aligned}$$

   More general problems can be put in this form. Suppose you are given the *maximization* problem:

$$\begin{aligned} \underset{x,y\in\mathbb{R}}{\text{maximize}} \quad & x+2y \\ \text{subject to} \quad & x^2+y^2=1. \end{aligned}$$

   Show how to write this problem in the standard form above. (I'm not asking you to solve the problem.)

2. Find the local and global minimizers of the following functions:
   (a) $f(x) = x^2 + 2x$.
   (b) $f(x) = x^2 e^{-x^2}$.
   (c) $f(x) = x + \sin x$.

3. Compute the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 4 & 0 \\ 3 & 6 \end{bmatrix}$.

4. (a) Find the Hessian matrix of the function $f(x,y) = e^{xy}$ (in terms of x and y).
   (b) Use it to classify the critical point $(0,0)$ as a (strict) local minimizer of $f(x,y)$, (strict) local maximizer of $f(x,y)$, or none of these.

5. *(Only 4-credit students need to do this problem.)*
   Let $\mathbf{x},\mathbf{y} \in \mathbb{R}^n$ be unit vectors: $\|\mathbf{x}\| = \|\mathbf{y}\| = 1$.
   Prove that $\mathbf{x}+\mathbf{y}$ is orthogonal to $\mathbf{x}-\mathbf{y}$.

General instructions for writing up homework:

- *If you're taking the class for 4 credits (as opposed to the default of 3), write this on your assignment so that it can be graded appropriately.*
- *When writing up solutions, if you use a result from your textbook, say the result you're using (by name, or theorem number, or whatever) and why it applies. E.g., "So $f''(5) = 1$. By the second derivative test, since $f''(5) > 0$, the critical point $x = 5$ is a strict local minimizer."*
- *You don't need to show your work for routine computations, but if you get those wrong without showing your work, you'll miss the opportunity for partial credit.*
- *Write proofs in complete sentences.*