1. On the first day of class, we discussed optimization problems in the standard form

\[
\begin{align*}
\text{minimize} & \quad f(x_1, x_2, \ldots, x_n) \\
\text{subject to} & \quad g_1(x_1, x_2, \ldots, x_n) \leq 0, \\
& \quad \ldots, \\
& \quad g_m(x_1, x_2, \ldots, x_n) \leq 0.
\end{align*}
\]

More general problems can be put in this form. Suppose you are given the maximization problem:

\[
\begin{align*}
\text{maximize} & \quad x + 2y \\
\text{subject to} & \quad x^2 + y^2 = 1.
\end{align*}
\]

Show how to write this problem in the standard form above. (I’m not asking you to solve the problem.)

2. Find the local and global minimizers of the following functions:
   (a) \( f(x) = x^2 + 2x \).
   (b) \( f(x) = x^2 e^{-x^2} \).
   (c) \( f(x) = x + \sin x \).

3. Compute the eigenvalues and eigenvectors of the matrix \[
\begin{bmatrix}
4 & 0 \\
3 & 6
\end{bmatrix}
\].

4. (a) Find the Hessian matrix of the function \( f(x, y) = e^{xy} \) (in terms of \( x \) and \( y \)).
   (b) Use it to classify the critical point \((0, 0)\) as a (strict) local minimizer of \( f(x, y) \), (strict) local maximizer of \( f(x, y) \), or none of these.

5. (Only 4-credit students need to do this problem.)
   Let \( x, y \in \mathbb{R}^n \) be unit vectors: \( \|x\| = \|y\| = 1 \).
   Prove that \( x + y \) is orthogonal to \( x - y \).

General instructions for writing up homework:

- If you’re taking the class for 4 credits (as opposed to the default of 3), write this on your assignment so that it can be graded appropriately.
- When writing up solutions, if you use a result from your textbook, say the result you’re using (by name, or theorem number, or whatever) and why it applies. E.g., “So \( f''(5) = 1 \). By the second derivative test, since \( f''(5) > 0 \), the critical point \( x = 5 \) is a strict local minimizer.”
- You don’t need to show your work for routine computations, but if you get those wrong without showing your work, you’ll miss the opportunity for partial credit.
- Write proofs in complete sentences.