Math 482: Linear Programming — Mikhail Lavrov

Homework #4

Spring 2020 — *Due Friday, February 28*

1. Write down the dual of the linear program below. (Do not solve).

$$\begin{aligned} \underset{x,y,z\in\mathbb{R}}{\text{maximize}} \quad & x + y + z \\ \text{subject to} \quad & 2x + y + 2z \le 14 \\ & x + z \le 8 \\ & 2x + 2y - z \le 18 \\ & x, y, z \ge 0. \end{aligned}$$

2. Determine whether $(x, y, z) = (5, 4, 0)$ is the optimal solution to the linear program from problem 1, using complementary slackness.

3. Consider the problem below:

$$\begin{aligned} \underset{\mathbf{x}\in\mathbb{R}^n}{\text{maximize}} \quad & c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ \text{subject to} \quad & a_1x_1 + a_2x_2 + \cdots + a_nx_n \le 1, \\ & x_1, x_2, \dots, x_n \ge 0. \end{aligned}$$

   Assume that $a_1, \dots, a_n, c_1, \dots, c_n > 0$.

   (a) Write down the dual program.

   (b) Determine the optimal dual solution. (This will of course depend on $a_1, \dots, a_n$ and $c_1, \dots, c_n$, but you should describe how.)

   (c) Find a primal solution with the same objective value.

4. Use the simplex method to solve the linear program below. Then, use your final simplex tableau to find the optimal dual solution.

$$\begin{aligned} \underset{x,y\in\mathbb{R}}{\text{maximize}} \quad & x - y + z \\ \text{subject to} \quad & x + 2y + z \le 5 \\ & 2x + y + z \le 6 \\ & x, y, z \ge 0. \end{aligned}$$

5. *(Only 4-credit students need to do this problem.)*

Consider the following linear program discussed in class:

$$
\begin{aligned}
\underset{\mathbf{x}\in\mathbb{R}^d}{\text{maximize}} \quad & x_d \\
\text{subject to} \quad & 0.1 \le x_1 \le 1 - 0.1, \\
& 0.1x_1 \le x_2 \le 1 - 0.1x_1, \\
& \dots \\
& 0.1x_{d-1} \le x_d \le 1 - 0.1x_{d-1}, \\
& x_1, x_2, \dots, x_d \ge 0.
\end{aligned}
$$

Let $\mathcal{P}_d$ be the "terrible trajectory"—the path between adjacent basic feasible solutions defined recursively as follows:

- $\mathcal{P}_1$ starts at $(0, 0, 0)$ and increases x_1 from its lower bound to its upper bound;
- $\mathcal{P}_k$ follows $\mathcal{P}_{k-1}$, then increases x_k from its lower bound to its upper bound, then undoes the steps of $\mathcal{P}_{k-1}$ in reverse order.

Show that the objective value increases with every step along $\mathcal{P}_d$. (Induct on d.)