Math 482: Linear Programming	Mikhail Lavrov
Homework #3	
Spring 2020	*Due Monday, February 17*

1. Let $P \subseteq \mathbb{R}^3$ be the convex polyhedron with only the following four extreme points: $(0,0,0)$, $(0,1,1)$, $(1,0,1)$, and $(1,1,0)$.
   (a) Write down a set of four linear inequalities describing P.
   (b) Show directly from the definition that the point $(\frac{2}{3}, \frac{2}{3}, \frac{2}{3})$ is *not* an extreme point of P.

2. Use the two-phase simplex method to solve the following linear program:

$$\begin{aligned} \underset{x_1,x_2,x_3\in\mathbb{R}}{\text{maximize}} \quad & x_1 + x_2 + 3x_3 \\ \text{subject to} \quad & 2x_1 + x_3 = 2 \\ & x_2 + x_3 = 3 \\ & 4x_1 + x_2 + 3x_3 = 7 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

3. Use lexicographic pivoting to solve the following linear program:

$$\begin{aligned} \underset{x,y\in\mathbb{R}}{\text{maximize}} \quad & x - y \\ \text{subject to} \quad & x - 2y \leq 0 \\ & x - 3y \leq 0 \\ & y \leq 3 \\ & x, y \geq 0 \end{aligned}$$

4. Consider the following linear program:

$$\begin{aligned} \underset{\mathbf{x}\in\mathbb{R}^{10}}{\text{maximize}} \quad & x_1 + 2x_2 + 3x_3 + 4x_4 + 2x_5 - x_6 + 4x_7 + 4x_8 + 2x_9 - x_{10} \\ \text{subject to} \quad & x_1 + x_2 - x_3 + 2x_4 - x_5 + 3x_6 + 2x_7 - x_8 + x_9 + 2x_{10} = 3 \\ & 3x_1 + 4x_2 + 2x_3 + 7x_4 + 5x_5 + 6x_6 - 2x_7 + 9x_8 + 8x_9 + 9x_{10} = 10 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \geq 0. \end{aligned}$$

   (a) Starting with basic variables $\mathcal{B} = (x_1, x_2)$, compute the inverse matrix $A_{\mathcal{B}}^{-1}$ and the basic feasible solution corresponding to $\mathcal{B}$.
   (b) Perform one iteration of the revised simplex method from the basic feasible solution you found in part (a). Use Bland's rule for pivoting.

   Your answer should give the new basis $\mathcal{B}$, the new inverse matrix $A_{\mathcal{B}}^{-1}$, and the new basic feasible solution.

5. *(Only 4-credit students need to do this problem.)*

Your friend was solving a linear program with two inequality constraints on the variables x and y, as well as the nonnegativity constraints $x, y \geq 0$. After adding slack variables s_1, s_2 to deal with the constraints, your friend used the simplex method to arrive at the following tableau:

	x	y	s_1	s_2	
x	1	2	0	0	3
s_2	0	1	-1	1	1
$-z$	0	-2	-1	0	-4

Show that your friend must have made a mistake: there is no linear program of the form described which can result in this final tableau.

(Hint: what would the starting tableau have been?)