

## Homework #1

Spring 2020

Due Friday, January 31

1. Write a linear program for the following problem. (Do not solve.)

A ship is transporting rice and wheat from California to Alaska. It has three cargo holds with the following capacities:

- The forward cargo hold can carry at most 10,000 tons, and at most 400,000 cubic feet.
- The middle cargo hold can carry at 5,000 tons, and at most 250,000 cubic feet.
- The aft cargo hold can carry at most 12,000 tons, and at most 600,000 cubic feet.

In addition, for the ship to be balanced, each cargo hold must be filled to the same fraction of its total capacity, with respect to tonnage.

A ton of wheat takes up 44.7 cubic feet and can be sold at a profit of \$20; a ton of rice takes up 40.9 cubic feet and can be sold at a profit of \$18.

The goal is to maximize the profit from the ship's cargo.

2. Draw the feasible region for this linear program, then solve it using the naive approach.

$$\begin{aligned} & \underset{x,y \in \mathbb{R}}{\text{maximize}} && x + y \\ & \text{subject to} && 6x + 5y \leq 19, \\ & && y \leq 4x + 9, \\ & && 2x - 7y \leq 15. \end{aligned}$$

3. Convert the linear program below to equational form.

$$\begin{aligned} & \underset{x,y \in \mathbb{R}}{\text{minimize}} && x - 2y \\ & \text{subject to} && x + y \leq 10, \\ & && y \geq 0. \end{aligned}$$

4. Use the simplex method to solve the *minimization* problem given in the tableau below:

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$x_1$	1	0	2	-1	0	4
$x_2$	0	1	2	3	4	9
$-z$	2	0	3	-1	1	-7

5. (Only 4-credit students need to do this problem.)

- Rewrite the constraint  $|x| + |y| \leq 5$  as a combination of linear constraints.
- Show that there is no way to rewrite the constraint  $|x| + |y| \geq 5$  as a combination of linear constraints.