The first exam will, broadly speaking, cover all the material covered in class through Monday, September 16th. This includes general intro material to linear programs, and most topics related to the simplex method.

Below I try to summarize the important things we’ve covered that will be on the exam. It’s possible that I’ve missed a few topics, and it’s hard to summarize three weeks of content in two pages anyway; if you’re not sure about something, ask me.

1 Things you should know

Here are the definitions you should know:

- The objective value, constraints, feasible solutions, optimal solutions, and feasible region of a linear program.
- Convex sets, vertices, and extreme points.
- The basic variables, nonbasic variables, basic feasible solution, reduced costs of a simplex method tableau.
- Slack variables and (for the two-phase simplex method) artificial variables.
- Unbounded and inconsistent/infeasible linear programs.
- Degenerate pivots, cycling, and pivot rules.
- Specific pivot rules: the largest/most negative reduced cost rule, Bland’s rule, lexicographic pivoting.

You should know and understand the following results about linear programs:

- If the feasible region of a linear program is nonempty and bounded, then the linear program has an optimal solution, and at least one optimal solution is a basic feasible solution.
- The vertices/extreme points of the feasible region are precisely the basic feasible solutions of the linear program.
- In non-degenerate cases, the simplex method is guaranteed to produce better and better solutions, and when it stops, it finds an optimal solution.
- Both Bland’s rule and the lexicographic pivoting rule prevent cycling.
2 Things you should be able to do

Convert between different forms of a linear program:

- Turn inequalities into equations and equations into inequalities.
- Convert a program with unbounded variables to one with nonnegative variables.
- Switch between minimizing and maximizing.

Skills related to the simplex method. (You can probably summarize most of these as “be able to use the simplex method”, but maybe you’d appreciate specifics.)

- Read off the basic feasible solution and objective value in a tableau.
- Move between valid tableaux by pivoting on variables.
- Choose entering variables (based on the objective function) and leaving variables (based on the nonnegativity constraints).
- Notice when a tableau represents an optimal solution.
- Notice when a linear program is unbounded.
- For linear programs with constraints \( \{Ax \leq b, x \geq 0\} \) in which \( x = 0 \) is feasible, use the slack variables to write down an initial feasible solution.
- For general linear programs, use the two-phase simplex method to find an initial feasible solution.
- When using the two-phase simplex method, notice when the linear program is infeasible.
- Use the most negative reduced cost rule (when minimizing) to choose entering variables.
- Use Bland’s rule to choose entering and leaving variables.
- Use the lexicographic pivoting rule (introducing \( \epsilon_1, \ldots, \epsilon_m \)) to choose leaving variables.

In general, unless I specify a pivoting rule I want you to use, you can make any (valid) choices of entering and leaving variables you like when solving a linear program.