1. Show that any $n \times n$ matrix following the pattern

$$
\begin{bmatrix}
1 & 0 & 1 & \cdots & 0 & 1 \\
0 & 1 & 0 & \cdots & 1 & 0 \\
1 & 0 & 1 & \cdots & 0 & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 1 & 0 & \cdots & 1 & 0 \\
1 & 0 & 1 & \cdots & 0 & 1
\end{bmatrix}
$$

is totally unimodular: any submatrix obtained by taking any $k$ rows and any $k$ columns has determinant $-1$, $0$, or $1$.

(*It may help to consider the cases $k \leq 2$ and $k \geq 3$ separately.*)

2. An *independent set* in a graph is a set $S$ of vertices such that no edge has both endpoints in $S$.

Write down a linear program encoding the problem of finding a largest independent set in the graph below.

3. Given the network below, with label $x/y$ denoting a flow of $x$ and a total capacity of $y$ along an edge, draw the residual graph, and use it to list all possible augmenting paths.

4. Find examples of networks with the following properties:

   (a) A network with a unique maximum flow, but multiple minimum cuts.
(b) A network with multiple maximum flows, but a unique minimum cut.
(c) A network with multiple maximum flows and multiple minimum cuts.
For each example, describe the maximum flow(s) and the minimum cut(s).

5. (Only 4-credit students need to do this problem.)
   For the linear program you write down in problem 2, write down the dual linear program and interpret it as a combinatorial optimization problem.