1. Use phase one of the two-phase simplex method to determine if the system of equations
\[
\begin{align*}
3x_1 + 2x_2 - x_3 &= 1 \\
x_1 - x_3 &= 1
\end{align*}
\]
has a solution in which \(x_1, x_2, x_3 \geq 0\).

2. Use lexicographic pivoting to solve the following linear program:

\[
\begin{align*}
\text{maximize} \quad & x - y \\
\text{subject to} \quad & x - 2y \leq 0 \\
& x - 3y \leq 0 \\
& y \leq 3 \\
& x, y \geq 0
\end{align*}
\]

3. Consider the following linear program:

\[
\begin{align*}
\text{maximize} \quad & x_1 + 2x_2 + 3x_3 + 4x_4 + 2x_5 - x_6 + 4x_7 + 4x_8 + 2x_9 - x_{10} \\
\text{subject to} \quad & x_1 + x_2 - x_3 + 2x_4 - x_5 + 3x_6 + 2x_7 - x_8 + x_9 + 2x_{10} = 3 \\
& 3x_1 + 4x_2 + 2x_3 + 7x_4 + 5x_5 + 6x_6 - 2x_7 + 9x_8 + 8x_9 + 9x_{10} = 10 \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \geq 0.
\end{align*}
\]

(a) Starting with basic variables \(B = (x_1, x_2)\), compute the inverse matrix \(A_B^{-1}\) and the basic feasible solution corresponding to \(B\).

(b) Use the revised simplex method to pick the first variable to pivot on (according to Bland’s rule); update \(B\), \(A_B^{-1}\), and the basic feasible solution accordingly. (You should only do one step of the revised simplex method.)

4. Write down the dual of the linear program below. (Do not solve).

\[
\begin{align*}
\text{minimize} \quad & x + y + z \\
\text{subject to} \quad & x + 2y \leq 4 \\
& x - y - z = -1 \\
& 2x + y + z \geq 2 \\
& x, y \geq 0 \text{ (}z\text{ unrestricted)}
\end{align*}
\]
5. *(Only 4-credit students need to do this problem.)*\(^1\)

Consider the following linear program discussed in class:

\[
\begin{align*}
\text{maximize} & \quad x_d \\
\text{subject to} & \quad 0.1 \leq x_1 \leq 1 - 0.1, \\
& \quad 0.1x_1 \leq x_2 \leq 1 - 0.1x_1, \\
& \quad \ldots \\
& \quad 0.1x_{d-1} \leq x_d \leq 1 - 0.1x_{d-1}, \\
& \quad x_1, x_2, \ldots, x_d \geq 0.
\end{align*}
\]

Let \( P_d \) be the “terrible trajectory”—the path between adjacent basic feasible solutions defined recursively as follows:

- \( P_1 \) starts at \((0, 0, 0)\) and increases \( x_1 \) from its lower bound to its upper bound;
- \( P_k \) follows \( P_{k-1} \), then increases \( x_k \) from its lower bound to its upper bound, then undoes the steps of \( P_{k-1} \) in reverse order.

Show that the objective value increases with every step along \( P_d \). (Induct on \( d \).)

\(^1\)This is a long problem statement, so ask me after class or during office hours if it is not clear.