Math 412: Topics Covered in Exam 3

Misha Lavrov

April 13, 2018

The second exam will, broadly speaking, cover all the material covered in class from Wednesday, March 7th to Wednesday, April 11th.

Here are some of the major topics in each section of the book, as well as some interesting exercises from each section. I also recommend reviewing the homework assignments, especially any questions that you didn’t solve correctly.

4 Connectivity and Paths

4.1 Cuts and connectivity

There’s a bunch of definitions here, which you should be comfortable with. Theorem 4.1.9 gives some basic bounds on the edge and vertex connectivity of a graph which you should know and understand.

Some interesting problems from this section: 10, 12, 14, 24.

4.2 $k$-connected Graphs

There are several characterizations of 2-connected graphs, which you should be familiar with; most important of them is ear decomposition.

For $k$-connected and $k$-edge-connected graphs, the most important thing to know is Menger’s theorem.

Some interesting problems from this section: 8, 9, 11, 20, 23, 24, 26.

4.3 Network Flow Problems

There are several important definitions here: a network, a feasible flow in a network and its value, an $s,t$-cut in a network and its capacity, maximum flows, and minimum cuts.

The maximum flow-minimum cut theorem is important, and you should understand its applications to proving other theorems. You should know how an augmenting path can be used to increase the
value of a flow, and how the residual graph of a network flow can be used to find an augmenting path or a minimum cut.

Some interesting problems from this section: 1, 2, 3, 5, 7, 10, 13.

6 Planar Graphs

6.1 Embeddings and Euler’s Formula

This section includes basic definitions. Key results are Euler’s formula, its corollary for the upper bound on the number of edges in a planar graph, and the face length sum formula.

Some interesting problems from this section: 13, 18, 25, 27, 29, 30.

6.2 Characterization of Planar Graphs

You should understand the notion of a graph subdivision and of a graph minor, and know and understand the statements of Kuratowski’s theorem and Wagner’s theorem.

You should be able to use conflict graphs to determine if a graph is planar. I will not give you excessively complicated examples that require finding multiple conflict graphs and recursing on fragments, because you are not computer programs.

Some interesting problems from this section: 3, 4, 5, 11, 13. Problem 14 looks interesting but might be very hard.

6.3 Parameters of Planarity

Of the topics covered in this section of the book, we have only discussed coloring planar graphs. You should understand the idea behind coloring graphs (refer to section 5.1 of the textbook for definitions) and the proof that all planar graphs can be colored with five colors.

Some interesting problems from this section: 2, 3, 5, 12.