Transition-hw8

Due date: November 8

(1) Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. Show that the sequence of polynomials

$$p_n(t) = \sum_{k=0}^{n} \frac{t^k}{k!} A^k$$

converges pointwise in $\mathbb{R}^9$ and calculate the limit. Also find the JNF for A.

(2) Do the same as in 1) but now for $A = \begin{pmatrix} 4 & 1 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{pmatrix}$ and the series for $\cos(t) = \sum_{k=0}^{\infty} (-1)^k \frac{t^{2k}}{(2k)!}$. Also compute the JNF.

(3) A liner map $T : V \to V$ is called nilpotent, if there exists $m \in \mathbb{N}$ such that $T^m = 0$. Let A be an upper diagonal matrix and T_A the induced linear map on $\mathbb{C}^n$. A is called nilpotent if T_A is nilpotent. Characterize nilpotent upper diagonal matrices. Find a nilpotent 2×2 matrix with non-zero coefficients on the diagonal (hint similarity).

(4) Consider $V = C(\mathbb{R})$, $r > 0$ and $T : C(\mathbb{R}) \to C(\mathbb{R})$ given by $T(f)(t) = f(t + r)$. Calculate $e^T = \sum_{k=0}^{\infty} \frac{T^k}{k!}$ where convergence takes place pointwise. How would you define $T^{\frac{1}{2}}$ and e^{tT}?