Proof (Lemma 1) We know that

$$\mathcal{V}_p^0(X,Y)^* = \Pi_{p'}(Y, X^{**}) = \Pi_{p'}(Y,X)$$

Hence

$$(\mathcal{V}_p^0)^{**}(X,Y) = \mathcal{I}_p(X, Y^{**})$$

In particular

$$\mathcal{V}_p^0(T) = \mathcal{I}_p(T : X \to Y^{**}) = \mathcal{V}_p^0(T : X \to Y^{**})$$

$\square$

Remark: The proof we presented in class does not exactly calculate

$$\Pi_{p'}(Y, X)^* = \mathcal{I}_p(X, Y^{**})$$

In general we may calculate

$$(\mathcal{F}(X,Y), \Pi_{p'})^* = \mathcal{I}_{p'}(Y, X^{**})$$

Proof: We consider the collection (G, F)

$$G \subseteq X^\vee \quad F \subset Y$$

For every such pair

$$\pi_p(G^\vee, F) \subseteq \pi_p(X, F)$$

and hence $\varphi : (F(X,Y), \pi_p) \to \mathbb{K}$ of norm 1

gives a functional on $\pi_p(G^\vee, F)$ and hence

$$\begin{tikzcd}
X \arrow[d, phantom, "\cap"] & \supset F \arrow[r, "S_\varphi"] \arrow[d] & G^\vee & X \arrow[l, two heads] \\
C(B_X) \arrow[r] & \ell_p^m \arrow[r, "D_\sigma"] & \ell_{p'}^m \arrow[u, "R"] &
\end{tikzcd}$$

As before we extend the picture, and define the weak*-limit

$$\hat{S} : C(B_{X^\vee}) \longrightarrow Y^{**}$$

$$\langle \hat{S}(f), g \rangle = \lim_{(F,G)\; g \in G} \langle S_{FG}(f), g \rangle$$

Then $\pi_{p'}(\hat{S}) \leq 1$, and it is easy to see that

$$\langle \hat{S}(f), g \rangle = \varphi(g \otimes f) \qquad \forall f \in X, g \in Y^\vee$$